STABILITY STUDIES OF VARIABLE SPEED INDUCTION MACHINE

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CERTIFICATE

This is to certify that the thesis entitled, 'Stability Studies of Variable Speed Induction Machine' is a record of the work carried out under our supervision and that it has not been submitted elsewhere for a degree.

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ABSTRACT

The work presented here deals with the stability studies of a variable speed induction machine. The needs for this study are identified. The machine model is developed. The regions in which the machine exhibits unstable operation are established. The nonlinear model of the machine is digitally simulated to investigate the machine performance at various operating points to establish the fact that the machine oscillates in the instable region of operation.

A short cut method is developed to estimate the oscillation frequency with fair accuracy. In the end suggestion is given to estimate oscillation amplitude on similar lines.

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CHAPTER 1

INTRODUCTION

1.o SPEED CONTROL OF INDUCTION MOTOR

The importance of induction motor , as a drive for various systems , in an industry is well known.

The very prominent advantage is that this machine is relatively inexpensive and rugged. In the past the machine had the disadvantage that its speed was not easily adjustable.

The introduction of static , variable frequency , variable voltage , inverters employing the thyristors has made the use of variable speed induction machine , technically attractive. The most common ways of controlling the speed of induction motors are the following :

a) insertion of resistance into the rotor circuit
b) pole changing
c) varying the supply frequency
d) cascade connection of induction motors with other machines.

Apart from the methods mentioned above , there are a variety of special methods such as control by means of saturable reactors , control by pulsed supply etc. Due to the large energy losses involved , the speed control of

induction motors with resistance inserted into the rotor circuit is not economical for continuous operation and control at constant torque.

Speed control by pole changing gives the motor a corresponding number of fixed speeds whose values depends on the supply frequency and the number of pole pairs set up in the winding. For example, four-speed motors for 50 cycle supply are manufactured for the following series of speeds (synchronous), 3000/1500/1000/500; 3000/1500/750/375; 1500/1000/750/500; 1000/750/500/375. From the speed series given above, it is seen that the speed control range is of the order of 6 :1 to 8 :1. It is found impracticable to increase this range because for a synchronous speed of 375 rpm it will be necessary to design a motor of prohibitive size.

Pole changing can only be applied where there is no need for smooth speed control because its speed control is stepped. Adjustable-frequency speed control is advantageous in that it provides a relatively wide speed control range of 10 :1 to 12 :1 and high smoothness of control. The speed-torque characteristics of the induction motor will have ample stiffness and therefore ensure stable operation. If the magnetic field of the induction motor is maintained unchanged, speed control will be accomplished at constant torque.

The basic possibility of speed control with variable supply frequency is indicated by the synchronous speed equation $\omega_o = 2\pi f_L/P$. To provide an adjustable frequency supply, use is made of a special generator or frequency changer serving to feed a single motor or a group of induction motors operating under identical conditions. Typical examples of such group drives are mill transfer roll tables, textile machines, certain kinds of conveyer installations, and other equipment.

In the case of the mill roll tables, each roll is individually driven by a squirrel-cage motor having a rating of several kilowatts. As many as hundred or more of these motors will be group operated at modern steel mills. In varying the supply frequency, it is necessary to make the characteristics retain high stiffness throughout the entire range of speed control and allow the motors to have adequate overload torque capacity. This may be achieved by causing each motor to operate with its magnetic flux maintained constant. For the induction motor, it may be assumed that the product $f_1\phi$ is directly proportional to the applied voltage v_1 as a first approximation. To maintain the magnetic flux unchanged, it is hence necessary to control the frequency so that the ratio

$v_1/f_1 = \phi =$ const

In the frequency method of speed control the speed-torque characteristics retain relatively high stiffness, while the maximum torque at higher frequencies remains practically unchanged. Only at the lowest frequencies, due to relative rise in the influence of the voltage drop in the stator, the magnetic field undergoes a significant decrease in strength. As a consequence, the maximum torque drops to a lower value. In many applications it may be necessary to operate these drives at 5% or 15% of the rated speed of the machines. To facilitate design, analytical methods have been formulated to accurately predict system performance over this range of operating frequencies.

1.1 INSTABILITY IN THE MOTOR

In the literature it has been shown that during low-frequency operation, the reluctance-synchronous machine as well as the synchronous machine exibits continuous oscillations in speed and in some cases may evenbecome unstable. These regions of instability occur even with balanced sinusoidal applied stator voltages. Hence instability of machines having a symmetrical rotor windings is not necessarily due to the unbalanced system voltages.

Robert [2] has shown that an induction motor is also lightly damped at low frequencies when operating from a balanced

set of sinusoidal voltages. Recent studies have also indicated that variable speed induction motor drives may become unstable due to the unbalanced system voltages. The wide range of operating voltages and frequencies encountered in such drives leads to effects which do not occur at fixed 50 Hz operation. One such effect , the occurrence of a continuous steady-state speed oscillations in induction motor drives , is the subject of this thesis The region where the machine exibits such phenomena is called the instable zone , and here , although not widely recognized, machine does not come to a stand-still as happens in the case of a synchronous machine.

In general instable operation of a induction m/c means sustained oscillations. For a given machine load parameters , this region of instability depends on the inverter voltage and frequency which are the control variables in the drive.

1.2 SCOPE OF THE THESIS

In Chapter 3 a method to find out this region is described. Digital simulation of induction machine is done to investigate its behaviour in this region. Machine operation is studied around the unstable operating points. Keeping all other machine parameters constant the sustained oscillations are obtained for different operating frequencies.

Also similar studies are carried out with different rotor resistance , stator resistance , rotor inertias and various loadings. Machine model for these studies is discussed in Chapter 2.

In Chapter 5 a short cut method is given to find out the frequency of the oscillations , which is simple to carry out analytically as well as digitally. The results obtained from this method are compared with those obtained from the non-linear model analysis.

In the end suggestions are given to estimate amplitude of the oscillations on similar lines. The studies carried on the digital computer are presented in a very useful and condensed manner.

CHAPTER 2

PERFORMANCE EQUATION OF THE MACHINE

2.0 INTRODUCTION

In establishing the equations which describe the behaviour of induction machinery, it is generally sufficient to consider an elementary 2 pole, 2 phase symmetrical machine.

This development can then be extended to include a machine having any number of poles by simply multiplying the expression for torque by the number of pole pairs.

As we are only interested in balanced conditions, the modifications necessary to include 3 phase machine are equally straight forward. If, however, unbalanced or unsymmetrical operation is to be analysed, it becomes necessary to consider the 2 phase and 3 phase machine individually.

A symmetrical machine is generally defined as a polyphase machine with

i) Uniform air gap

ii) Linear magnetic circuits

iii) Identical stator windings, distributed so as to produce a sinusoidal MMF wave in space with the phases, and arranged so that only one rotating MMF wave is established by balanced stator currents.

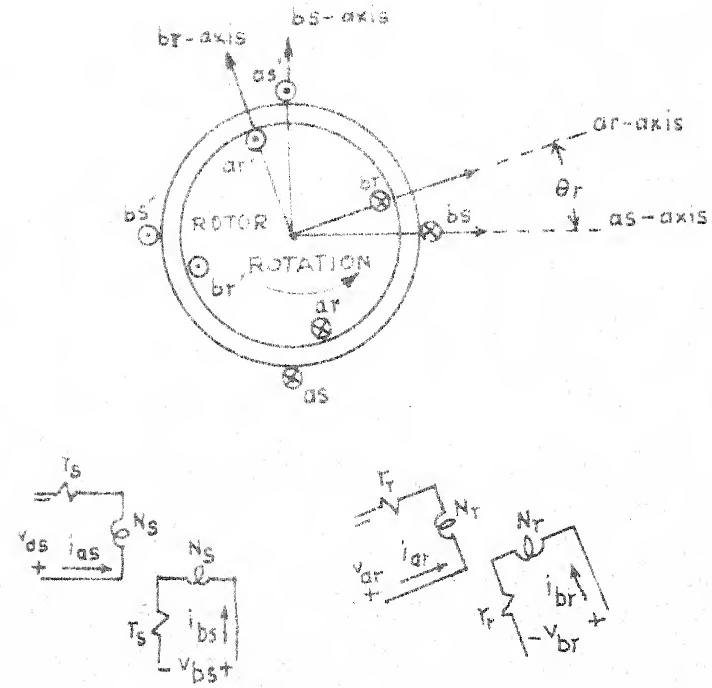

FIG. 2.1 A 2-POLE 2-PHASE SYMMETRICAL MACHINE

iv) Rotor coils or bars are arranged so that, for any fixed time, the rotor MMF wave can be considered to be a space sinusoid having the same number of poles as the stator MMF wave.

Although the symmetrical machine is an idealised machine, it offers a means of predicting the performance of many types of polyphase induction machine. Some important factors whibh effect the performance of the actual machine but have been neglected in the symmetrical machine model are:

i) Nonlinear magnetic circuit

ii) Change in resistance due to temperature and frequency changes.

iii) Harmonic content of the MMF wave due to nonideal winding distribution.

2.1 MODELLING OF A 3-PHASE SYMMETRICAL INDUCTION MACHINE

As a first step towards modelling of a 3 phase symmetrical induction machine, a 2 phase symmetrical machine is taken. The simplified representation is shown in Fig.2.1 will be used. When a stator winding is distributed for the purpose of producing a sinusoidal MMF wave in space, it is convenient to portray the windings as an equivalent single coil and express the mutual coupling between it and an equivalent rotor coil as a sinusoidal function of the angular displacement between their magnetic axis.

If the induction machine has either squirrel cage rotor or a wound rotor with the same number of poles as the stator, the rotor can be considered as having equivalent coils (shown in Figure 2.1). The modifications which are necessary to include a double cage rotor or a rotor wound with different number of phases than the stator, are straight forward and will not be considered here.

The stator windings are identical i.e. both windings have an identical number of turns N_s, identical resistance r_s, identical leakage inductance L_{ls} and identical self-inductance L_s. Similarly, equivalent rotor windings are identical which have the same effective turns N_r, resistance r_r, leakage inductance L_{lr} and self inductance L_r. The voltage equations for the stator phase are written

$$
\begin{aligned}
V_{as} &= p\lambda_{as} + i_{as} r_s \\
V_{bs} &= p\lambda_{bs} + i_{bs} r_s
\end{aligned}
\tag{2.1}
$$

In the case of rotor phase

$$
\begin{aligned}
V_{ar} &= p\lambda_{ar} + i_{ar} r_r \\
V_{br} &= p\lambda_{br} + i_{br} r_r
\end{aligned}
\tag{2.2}
$$

where λ is the total flux linkages and p is the operator d/dt. The flux-linkage equation can be written as

$$
\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{ar} \\ \lambda_{br} \end{bmatrix} = \begin{bmatrix} L_s & 0 & L_{sr}\cos\theta_r & -L_{sr}\sin\theta_r \\ 0 & L_{sr} & L_s\sin\theta_r & L_{sr}\cos\theta_r \\ L_{sr}\cos\theta_r & L_{sr}\sin\theta_r & L_r & 0 \\ L_{sr}\sin\theta_r & L_{sr}\cos\theta_r & 0 & L_r \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{ar} \\ i_{br} \end{bmatrix}
\tag{2.3}
$$

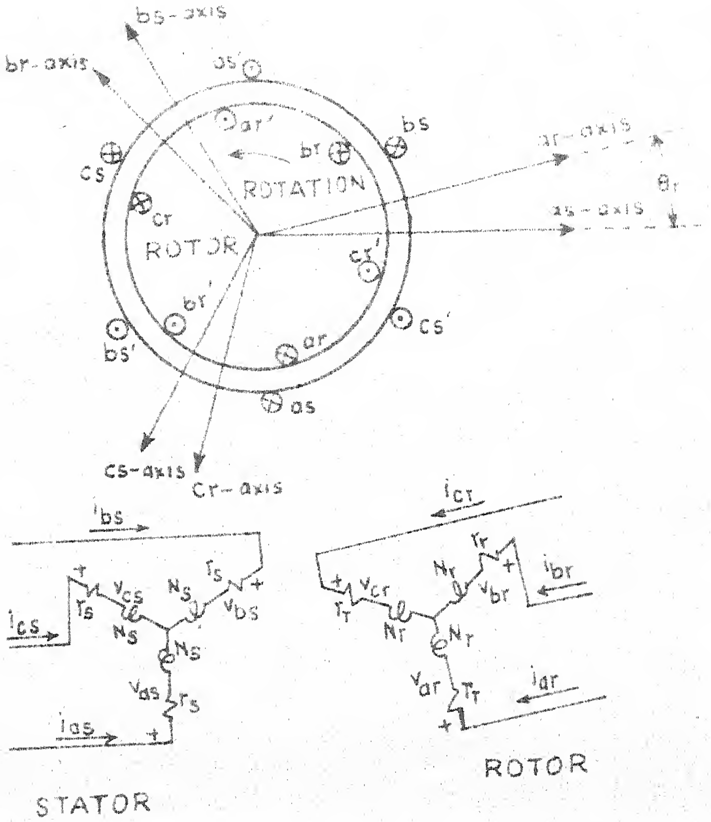

FIG. 2.2 A 2-POLE 3-PHASE SYMMETRICAL MACHINE

where L_{sr} is the amplitude of the mutual inductance between stator and rotor windings and θ_r is the angular displacement between the stator and rotor axis (Figure 2.1). Now extending this analysis to a 3 phase induction machine where winding configuration is as shown in Figure 2.2 we get the line-to-neutral stator voltages as

$$
\begin{aligned}
V_{as} &= p\lambda_{as} + i_{as}\, r_s \\
V_{bs} &= p\lambda_{bs} + i_{bs}\, r_s \\
V_{cs} &= p\lambda_{cs} + i_{cs}\, r_s
\end{aligned}
\tag{2.4}
$$

The line to neutral voltages are

$$
\begin{aligned}
V_{ar} &= p\lambda_{ar} + i_{ar}\, r_r \\
V_{br} &= p\lambda_{br} + i_{br}\, r_r \\
V_{cr} &= p\lambda_{cr} + i_{cr}\, r_r
\end{aligned}
$$

Since the stator and the rotor are 3-wire systems the flux linkages equations are given in (2.5).

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\end{aligned}
\qquad (2.4)
$$

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v_{br} &= p\lambda_{br} + i_{br} r_r \\
v_{cr} &= p\lambda_{cr} + i_{cr} r_r
\end{aligned}
$$

Since the stator and the rotor are 3-wire systems the flux linkages equations are given in (2.5).

$$
\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \\ \lambda_{ar} \\ \lambda_{br} \\ \lambda_{cr} \end{bmatrix}
=
\begin{bmatrix}
L_{ss} & 0 & 0 & L_{sr}\cos\theta_r & -L_{sr}\cos(\theta_r+2\pi/3) & L_{sr}\cos(\theta_r-2\pi/3) \\
0 & L_{ss} & 0 & L_{sr}\cos(\theta_r-2\pi/3) & L_{sr}\cos\theta_r & L_{sr}\cos(\theta_r+2\pi/3) \\
0 & 0 & L_{ss} & L_{sr}\cos(\theta_r+2\pi/3) & L_{sr}\cos(\theta_r-2\pi/3) & L_{sr}\cos\theta_r \\
L_{sr}\cos\theta_r & L_{sr}\cos(\theta_r-2\pi/3) & L_{sr}\cos(\theta_r+2\pi/3) & L_{rr} & 0 & 0 \\
L_{sr}\cos(\theta_r+2\pi/3) & L_{sr}\cos\theta_r & L_{sr}\cos(\theta_r-2\pi/3) & 0 & L_{rr} & 0 \\
L_{sr}\cos(\theta-2\pi/3) & L_{sr}\cos(\theta_r+2\pi/3) & L_{sr}\cos\theta_r & 0 & 0 & L_{rr}
\end{bmatrix}
\times
\begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \\ i_{ar} \\ i_{br} \\ i_{cr} \end{bmatrix}
\qquad (2.5)
$$

$$L_{ss} = L_s - L_{sm}$$
$$L_{rr} = L_r - L_{rm} \qquad (2.6)$$

where L_{sm} is the mutual between stator phases and L_{rm} is the mutual between motor phases.

2.2 TRANSFORMATION TO AN ARBITRARY REFERENCE FRAME

Due to the sinusoidal variation of mutual inductances with respect to the displacement angle θ_r, time-varying coefficients will appear in the voltage equations. This undesirable feature can be eliminated by a proper change of variables which, in effect, transforms the voltages and currents of both the stator and rotor to a common frame of reference. In most cases, the analysis of an induction machine is carried out in either a synchronously rotating reference frame or a stationary reference frame. It is, however, necessary to consider each reference frame separately in the development of the equations which describe the behaviour of the symmetrical machine. It is convenient therefore to develop the equations for an arbitrary reference frame and, from these general equations, obtain the equations for any specific reference frame.

The equations of transformation are expressions which formulate a change of variables and could be written without any physical interpretation. It is helpful, however, to correlate the change of variables (transformation equations)

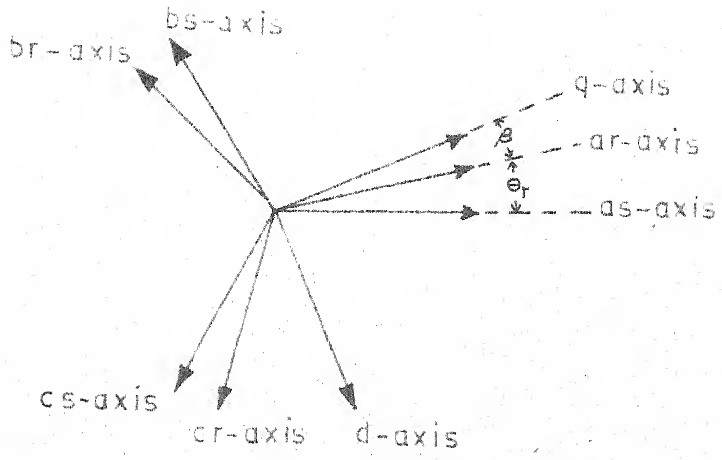


FIG. 2.3 AXES OF 2-POLE 3-PHASE SYMMETRICAL MACHINE

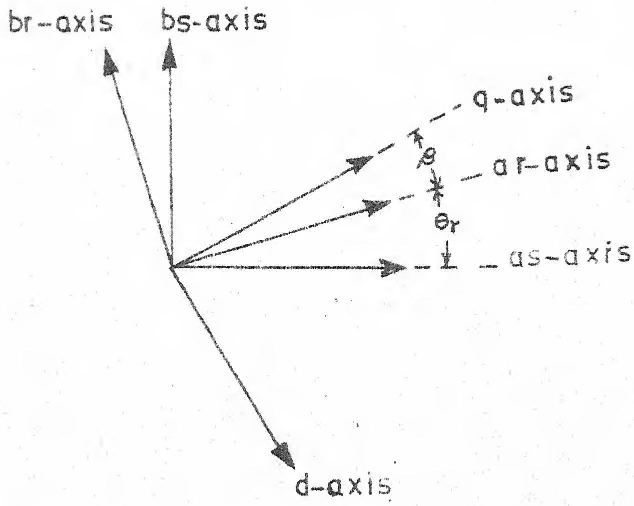


FIG. 2.4 AXES OF 2-POLE 2-PHASE SYMMETRICAL MACHINE

to trignometric relationships which exist between sets of axes.

To illustrate this facility, a third set of axes will be introduced. Figure 2.3 shows the angular relation of the stator and rotor axes of a 3-phase machine, with the third set which is an orthogonale set (d-q axis) rotating at an arbitrary electrical angular velocity w. It is clear that the as-bs set is fixed in the stator. The ar-br set is fixed in the rotor and hence rotates at an electrical angular velocity of w_r. The time-zero angular relationship between the three sets of axes can be selected arbitrarily. However, it is convenient to assume that at time zero, the q, a_r and a_s axes coincide.

The equations of transformation, which can be correlated to the angular relation of the axis in (Figure 2.3) are

STATOR

$$f_{qs} = \frac{2}{3}[f_{as} \cos\theta + f_{bs} \cos(\theta-2\pi/3) + f_{cs} \cos(\theta+2\pi/3)]$$

$$f_{ds} = \frac{2}{3}[f_{as} \sin\theta + f_{bs} \sin(\theta-2\pi/3) + f_{cs} \sin(\theta+2\pi/3)] \qquad (2.7)$$

$$f_{as} = \frac{1}{3}(f_{as} + f_{bs} + f_{cs})$$

ROTOR

$$f_{qr} = \frac{2}{3}[f_{ar}\cos\beta+f_{br}\cos(\beta-2\pi/3)+f_{cr}\cos(\beta+2\pi/3)]$$

$$f_{dr} = \frac{2}{3}[f_{ar}\sin\beta+f_{br}\sin(\beta-2\pi/3)+f_{cr}\sin(\beta+2\pi/3)] \qquad (2.8)$$

$$f_{ar} = \frac{1}{3}(f_{ar}+f_{br}+f_{cr})$$

where $\beta = \theta - \theta_r$ (2.9)

In these equations, the variable f can represent either voltage, current or flux-linkage. The equations of transformation are valid regardless of form of the voltages or current in either the stator or the rotor. However, the equations are restricted in that the instantaneous angular displacement θ of the arbitrary reference must be a continuous function.

The variables f_{os}, f_{or} are incorporated since in general three independent variables are necessary. When balanced conditions are considered, the three voltages and currents are defined by only two and the third substitute variable is unnecessary. If the transformation equations are used to transform the voltages and currents of both the stator and the rotor to the arbitrary reference frame, d-q axes the following equations are obtained.

$$
\begin{aligned}
V_{qs} &= p\lambda_{qs} + \lambda_{ds}p\theta + r_s i_{qs} \\
V_{ds} &= p\lambda_{ds} - \lambda_{qs}p\theta + r_s i_{ds} \\
V_{qr} &= p\lambda_{qr} + \lambda_{dr}p\beta + r_r i_{qr} \qquad (2.10) \\
V_{dr} &= p\lambda_{dr} - \lambda_{qr}p\beta + r_r i_{dr}
\end{aligned}
$$

where

$$\begin{aligned}
\lambda_{qs} &= L_{ss}\, i_{qs} + \frac{3}{2} L_{sr}\, i_{qr} \\
\lambda_{ds} &= L_{ss}\, i_{ds} + \frac{3}{2} L_{sr}\, i_{dr} \\
\lambda_{qr} &= L_{rr}\, i_{qr} + \frac{3}{2} L_{sr}\, i_{qs} \\
\lambda_{dr} &= L_{rr}\, i_{dr} + \frac{3}{2} L_{sr}\, i_{ds}
\end{aligned} \qquad (2.11)$$

Generally the machine parameters are measured with respect to stator windings therefore , it is convenient to refer all quantities to the stator side with rotor quantities referred to stator side and with the self inductances separated into leakage inductance and a magnetising inductance , the voltage equations for a three phase machine becomes.

$$\begin{aligned}
V_{qs} &= p\lambda_{qs} + \lambda_{ds}\, p\theta + r_s\, i_{qs} \\
V_{ds} &= p\lambda_{ds} - \lambda_{qs}\, p\theta + r_s\, i_{ds} \\
V_{qr}' &= p\lambda_{qr}' + \lambda_{dr}'\, p\beta + r_r'\, i_{qr}' \\
V_{dr}' &= p\lambda_{dr}' - \lambda_{qr}'\, p\beta + r_r'\, i_{dr}'
\end{aligned} \qquad (2.12)$$

whefe

$$\begin{aligned}
\lambda_{qs} &= L_{\ell s}\, i_{qs} + M(i_{qs}+i_{qr}') \\
\lambda_{ds} &= L_{\ell s}\, i_{ds} + M(i_{ds}+i_{dr}') \\
\lambda_{qr} &= L_{\ell r}'\, i_{qr}' + M(i_{qs}+i_{qr}') \\
\lambda_{dr} &= L_{\ell r}'\, i_{dr}' + M(i_{ds} + i_{dr}')
\end{aligned} \qquad (2.13)$$

In which

$$L_{\ell s} = L_{ss} - \frac{3}{2} L_{ms}$$

$$L_{\ell r}' = L_{rr}' - \frac{3}{2} L_{ms} \qquad (2.15)$$

$$M = \frac{3}{2} L_{ms}$$

Expressing the relationship of voltages and currents, given above in matrix form one gets the following

$$\begin{bmatrix} V_{qs} \\ V_{ds} \\ V_{qr}' \\ V_{dr}' \end{bmatrix} = \begin{bmatrix} r_s+L_{ss}p & \omega L_{ss} & Mp & \omega M \\ -\omega L_{ss} & r_s+L_{ss}p & -\omega M & Mp \\ Mp & (\omega-\omega_r)M & (r_r'+L_{rr}p) & (\omega-\omega_r)L_{rr} \\ -(\omega-\omega_r)M & Mp & -(\omega-\omega_r)L_{rr} & (r_s+L_{rr}'p) \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr}' \\ i_{dr}' \end{bmatrix}$$

In terms of reactance we can write the equations as follows

$$\begin{bmatrix} V_{qs} \\ V_{ds} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_s+(p/\omega_b)X_s & (\omega/\omega_b)X_s & (p/\omega_b)X_m & (\omega/\omega_b)X_m \\ -(\omega/\omega_b)X_s & r_s+(p/\omega_b)X_s & -(\omega/\omega_b)X_m & (p/\omega_b)X_m \\ (p/\omega_b)X_m & [(\omega-\omega_r)/\omega_b]X_m & r_r'+\frac{p}{\omega_b}X_r & [(\omega-\omega_r)/\omega_b]X_r' \\ -[(\omega-\omega_r)/\omega_b]X_m & (p/\omega_b)X_m & -[(\omega-\omega_r)/\omega_b]X_r' & [r_r'+pX_r']/\omega_b \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr}' \\ i_{dr}' \end{bmatrix}$$

(2.16)

$$X_s \quad = X_{\ell s} + X_m \tag{2.17}$$

$$X_r' \quad = X_{\ell r}' + X_m \tag{2.18}$$

where p is the operator d/dt

r_s - stator resistance

r_r' - motor resistance (referred to the stator windings)

$X_{\ell s}$ = stator leakage reactance

$X_{\ell r}'$ = rotor leakage reactance

X_m = magnetising reactance

The electrical angular velocity of the rotor can be denoted by ω_r. The reference frame rotates at a specified but arbitrary angular velocity ω. If the machine is excited by a balanced sinusoidal set of 3 phase voltages of angular velocity ω_e, the q and d axis voltages V_{qs} and V_{ds} are sinusoids of frequency $(\omega_e - \omega)$and in the steady state the four d-q axis currents assume this same frequency.

To perturb the solution about a steady state operating condition it is of interest to select the reference frame wherein the voltage and current variables assume constant (d.c.) values for steady state conditions. It is clear that this result will occur only if $\omega - \omega_e = 0$. Hence setting $\omega = \omega_e$ the preceding equations are feferred to the synchronously rotating reference frame wherein a set of orthogonal axes rotate at the angular velocity of the applied voltages.

The Equation (2.16) becomes

$$\begin{bmatrix} v_{qs}^{e} \\ v_{ds}^{e} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_s+(p/\omega_b)X_s & f_R X_s & (p/\omega_b)X_m & f_R X_m \\ -f_R X_s & r_s+(p/\omega_b)X_s & -f_R X_m & (p/\omega_b)X_m \\ (p/\omega_b)X_m & f_R S X_m & -r_r'+(p/\omega_b)X_r' & f_R S X_r' \\ -f_R S X_m & (p/\omega_b)X_m & -f_R S X_r' & -r_r'+(p/\omega_b)X_r' \end{bmatrix} \begin{bmatrix} i_{qs}^{e} \\ i_{ds}^{e} \\ i_{qr}'^{e} \\ i_{dr}'^{e} \end{bmatrix} \tag{2.19}$$

where $S = (\omega_e - \omega_r)/\omega_e$ (2.20)

The quantity $f_R = \omega_e/\omega_b$ (2.21)

is called the frequency ratio and may also be interpreted as the applied frequency expressed in per unit. The subscript e denotes variables expressed in the synchronously rotating reference frame. The electromagnetic torque expressed in per unit is

$$T_e = (X_m)(i_{qs}^{e}\, i_{dr}'^{e} - i_{ds}^{e}\, i_{qr}'^{e}) \tag{2.22}$$

In addition, the per unit equation expressing the electromechanical behaviour of the system is expressed as

$$T_e = (2H/\omega_b)\, p\omega_r + T_L$$

where the inertia constant H is expressed in seconds and defined as the ratio of the stored kinetic energy at base mechanical speed to the base power. T_L is the per unit load torque applied to the machine.

CHAPTER 3

DETERMINATION OF REGION OF INSTABILITY

In order to establish the region in which machine becomes unstable the local stability study of the system is investigated.

For this, equations can be simplified considerably by linearising them about a steady state operating point. This method of small displacements has historically been fruitful in the analysis of electric machinery. Because the resulting equations are linear, any of the conventional techniques may be used to establish system stability. For example the Routh test or the Nyquist stability criterion could be employed. In this paper a method analogous to the root locus method is used to establish system stability.

If all the variables in Eqns.(2.19), (2.20), (2.23) are allowed to change by a small amount about a steady state operating point and if the terms which describe the steady state mode of operation are eliminated, the resulting equations can be written as

$$\begin{bmatrix} \Delta v_{qs}^{e} \\ \Delta v_{ds}^{e} \\ 0 \\ 0 \\ \Delta T_L \end{bmatrix} = \begin{bmatrix} r_s+(p/\omega_b)X_s & f_R X_s & (p/\omega_b)X_m & f_R X_m & 0 \\ f_R X_s & r_s+(p/\omega_b)X_s & -f_R X_m & (p/\omega_b)X_m & 0 \\ (p/\omega_b)X_m & f_R S_o X_m & r_r'+(p/\omega_b)X_r' & f_R S_o X_r' & -(X_m i_{dso}^{e}+X_r' i_{dro}'^{e}) \\ -f_R S_o X_m & (p/\omega_b)X_m & -f_R S_o X_r' & r_r'+(p/\omega_b)X_r' & (X_m i_{qso}^{e}+X_r' i_{qro}'^{e}) \\ X_m i_{dro}'^{e} & -X_m i_{qro}'^{e} & -X_m i_{dso}^{e} & X_m i_{qso}^{e} & -2Hp \end{bmatrix} \begin{bmatrix} \Delta i_{qs}^{e} \\ \Delta i_{ds}^{e} \\ \Delta i_{dr}'^{e} \\ \Delta i_{qr}'^{e} \\ \Delta \omega_r/\omega_b \end{bmatrix} \tag{3.1}$$

where

$$S_o = (\omega_e - \omega_{ro})/\omega_e$$

The zero subscripted variables denote steady state operating point quantities. Equation (3.1) can be written more concisely in partitioned matrix notation as

$$\begin{bmatrix} \Delta V \\ \\ \Delta T_L \end{bmatrix} = \begin{bmatrix} R & V_{10} \\ \\ V_{20}^T & 0 \end{bmatrix} \begin{bmatrix} \Delta i \\ \\ \Delta\omega_r/\omega_b \end{bmatrix} + p/\omega_b \begin{bmatrix} X & 0 \\ \\ 0 & -2H\omega_b \end{bmatrix} \begin{bmatrix} \Delta i \\ \\ \Delta\omega_r/\omega_b \end{bmatrix} \tag{3.2}$$

where

$$\Delta V = \begin{bmatrix} \Delta V_{qs}^e \\ \Delta V_{ds}^e \\ 0 \\ 0 \end{bmatrix} \tag{3.3}$$

$$\Delta i = \begin{bmatrix} \Delta i_{qs}^e \\ \Delta i_{ds}^e \\ \Delta i_{qr}^e \\ \Delta i_{dr}^e \end{bmatrix} \tag{3.4}$$

$$V_{10} = \begin{bmatrix} 0 \\ 0 \\ -(X_m i_{dso}^e + X_r' i_{dro}'^e) \\ X_m i_{qso}^e + X_r' i_{qro}'^e \end{bmatrix} \tag{3.5}$$

$$V_{20} = \begin{bmatrix} X_m \, i_{dro}{}'^{e} \\ -X_m \, i_{qro}{}'^{e} \\ -X_m \, i_{dso}{}^{e} \\ X_m \, i_{qso}{}^{e} \end{bmatrix} \tag{3.6}$$

$$R = \begin{bmatrix} r_s & f_R X_s & 0 & f_R X_m \\ -f_R X_s & r_s & -f_R X_m & 0 \\ 0 & f_R S_o X_m & r_r' & f_R S_o X_r' \\ -f_R S_o X_m & 0 & -f_R S_o X_r' & r_r' \end{bmatrix} \tag{3.7}$$

$$X = \begin{bmatrix} X_s & 0 & X_m & 0 \\ 0 & X_s & 0 & X_m \\ X_m & 0 & X_r' & 0 \\ 0 & X_m & 0 & X_r' \end{bmatrix}$$

and 0 represents a 4x1 column vector of zeros. The superscript T denotes the transpose, the column vectors V_{10} and V_{20} denote quantities expressed in per unit voltage and are established from steady state operating conditions. Upon solving for the vector denoting the time derivatives of currents (3.2) can be expressed.

$$p/\omega_b \begin{bmatrix} \Delta i \\ \\ \Delta\omega_r/\omega_b \end{bmatrix} = \begin{bmatrix} -X^{-1} R & -X^{-1} V_{10} \\ \\ (1/2H\omega_b) V_{20}^T & 0 \end{bmatrix}$$

$$\begin{bmatrix} \Delta i \\ \\ \Delta\omega_r/\omega_b \end{bmatrix} + \begin{bmatrix} X^{-1} & 0 \\ \\ 0 & -1/2H\omega_b \end{bmatrix} \begin{bmatrix} \Delta V \\ \\ \Delta T_L \end{bmatrix} \qquad (3.9)$$

Equation (3.9) constitutes the vector-matrix differential equation of the linearized system. The column vector $[\Delta i^T, \Delta\omega_r/\omega_b]^T$ is the state vector which contains the set of linearized system state variables. The column vector $[\Delta V^T, \Delta T_L]^T$ represents the set of system forcing functions. If the forcing function vector is set equal to zero the solution of (3.9) is

$$\begin{bmatrix} \Delta i \\ \\ \Delta\omega_r/\omega_b \end{bmatrix} = e^{A\omega_b t} \begin{bmatrix} \Delta i(0) \\ \\ \Delta\omega_r(0)/\omega_b \end{bmatrix} \qquad (3.10)$$

where

$$A\omega_b = \omega_b \begin{bmatrix} -X^{-1}R & -X^{-1}V_{10} \\ \\ (1/2H\omega_b)V_{20}^T & 0 \end{bmatrix} \qquad (3.11)$$

and $[\Delta i(0)^T, \Delta\omega_r(0)/\omega_b]^T$ is an arbitrary set of initial conditions. The matrix exponential function e^{At} represents the unforced response of the system and is called the fundamental or state transition matrix of the system. Local-asymptotic stability is assured if all elements of the transition matrix approach zero asymptotically as time approaches infinity.

$e^{At} \rightarrow 0$ if all the roots of the characteristic equation of A have negative real parts. The roots of the characteristic equation are given by those values of $\lambda' = \lambda/\omega_b$ for which the determinant

$$\left| \lambda \begin{bmatrix} I & 0 \\ \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} X^{-1}R & X^{-1}V_{10} \\ \\ -(1/2H\omega_b)V_{20}^T & 0 \end{bmatrix} \right| = 0$$

(3.12)

The parameter λ' which has been introduced into (3.12) serves to normalize the roots of the characteristic equation to the base frequency ω_b. The roots of (3.12) may be computed by solving for the roots of the resulting polynomial equation in λ', or they may be calculated directly by determining the eigen values of the matrix A.

3.1 STABILITY STUDIES

Solution of the roots of (3.12) provides a simple mean of predicting the behaviour of an induction motor at any operating frequency and for any torque load. Roots λ' with negative real parts correspond to terms which decrease exponentially with time. Values of λ' having positive real part results in elements in the state transition matrix which increases exponentially in time. In this study, 60 Hz is assumed to be the rated frequency and the per unit system employed is based on operation at this frequency (ω_b = 377 radians/sec.).

A change in steady state operating speed (change in frequency) is easiky incorporated by appropriate changes in the value of f_R.

In order to investigate thoroughly the stability of the induction machine, it is necessary to find the roots of the characteristic equation, for all practical operating frequencies and loading conditions and for a range of machine parameters.

From this study the range of frequencies and loading for which one gets positive real parts of the eigen value can

be found. This range gives us the instability range.

In variable speed systems, the amplitude of the applied voltages is decreased as frequency decreases in order to avoid saturation of the machine. However, if the voltage is decreased as a linear function of frequency, the breakdown torque is reduced significantly at low frequencies. Because an increased percentage of the applied voltage is dropped across the stator resistance as frequency is reduced. In this study, as a simple means of i_R compensation the voltage required to produce rated flux linkage at rated load (T_L = 1.0 p.u.) and rated speed (f_R = 1.0) has been predetermined from steady state considerations. For the set of parameters the terminal voltage required to satisfy this constraint is V = 1.025 p.u. When operating from a variable frequency source, the terminal voltage has been adjusted so that at any frequency

$$V = V_k + f_R V_m \tag{3.13}$$

where V_R = 0.025 and V_m = 1.0 p.u. (Ref. No. 2).
From Eq.(3.13) it is clear that the constant factor V_k serves to compensate for the stator i_R drop. It can be noted that when f_R = 1.0, the terminal voltage V = 1.025.

Figures 3.1 to 3.3 are the results of such a study taken from reference 2.

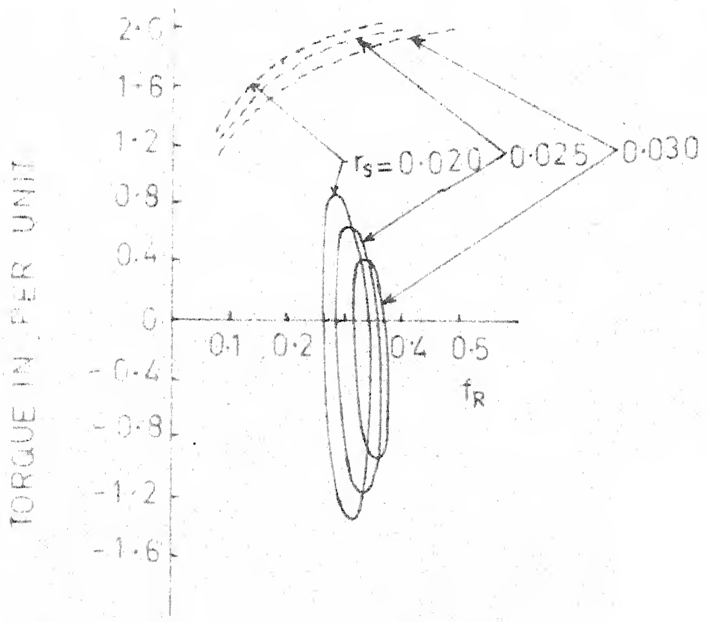


FIG. 3·1 REGIONS OF INSTABILITY FOR DIFFERENT VALUES OF STATOR RESISTANCE

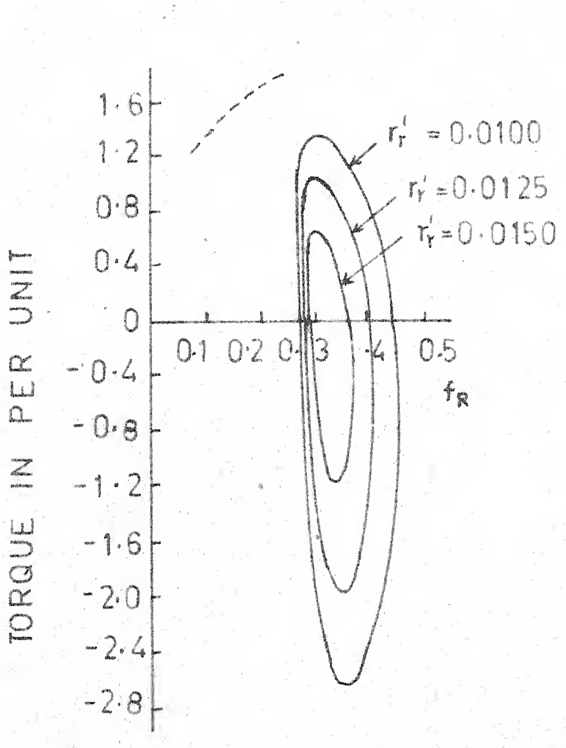


FIG. 3·2 REGIONS OF INSTABILITY FOR CHANGES IN ROTOR RESISTANCE

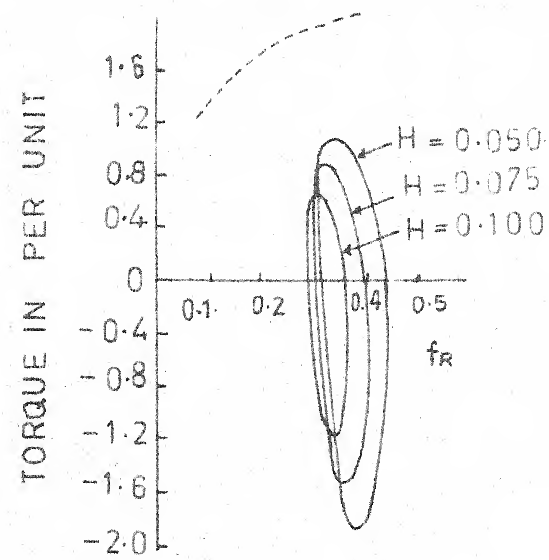


FIG.3·3 REGIONS OF INSTABILITY FOR DECREASE IN SYSTEM INERTIA

On the boundary of region of instability, one pair of conjugate complex eigen values has zero real part. The other pair has negative real part and the fifth eigen value is real and has negative value.

Following are the sets of eigen values on the boundary for a few Cases.

<u>Case 1</u>: Machine Parameters

$r_s = 0.025 /\ r_r' = 0.015 /\ X_{ls} = 0.1 / X_{lr}' = 0.1 /\ X_m = 3.5$

$H = 0.1 /\ V_k = 0.025 /\ V_m = 1.0$

Voltage vector

$$V = \begin{bmatrix} 0.315 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$T_L = 0.1$

$f_R = 0.290$

Slip, $S = 0.005$

Matrix,

$$w_b \underline{A} = \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & -0.71999 & 0.00468 & -0.00336 & 0.69396 \\ 0.00 & -0.01660 & -0.99929 & 0.03232 & 0.01033 \\ 0.00 & 0.69378 & -0.01906 & -0.00271 & 0.71993 \\ 0.00 & 0.00 & 0.03228 & 0.99947 & 0.00462 \end{bmatrix}$$

Eigen Values

1	-60.8756	+	j 120.5842
2	-60.8756	-	j 120.5842
3	0.0929	+	j 78.1368
4	0.0929	-	j 78.1368
5	-31.3585	+	j 0.00

Case 2

$T_L = 0.2, \quad S = 0.01$

Other machine parameters, same as case 1

$$w_b A = \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & -0.72147 & 0.00469 & -0.00357 & 0.69242 \\ 0.00 & -0.01666 & -0.99975 & 0.01060 & 0.01054 \\ 0.00 & 0.69225 & -0.01919 & -0.00414 & 0.72139 \\ 0.00 & 0.00047 & 0.01054 & 0.99993 & 0.00557 \end{bmatrix}$$

Eigen Values are:

1.	60.9170	+	j 120.5499
2	-60.9170	-	j 120.5499
3	0.0966	+	j 78.1800
4	0.0966	-	j 78.1800
5	-31.2831	+	0

Case 3

$T_L = 0.3, \quad S = .015$

Other machine parameters same as in case 1

$$w_b A = \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & -0.72357 & 0.00473 & -0.00357 & 0.69023 \\ 0.00 & -0.01675 & -0.99980 & -0.00284 & -0.01072 \\ 0.00 & 0.69005 & -0.01930 & -0.00547 & 0.72348 \\ 0.00 & 0.00293 & -0.00293 & 0.99997 & 0.00640 \end{bmatrix}$$

Eigen Values are

1.	-60.9453	+	j 121.2238
2	-60.9453	-	j 121.2238
3	0.0812	+	j 78.8939
4	0.0812	-	j 78.8939
5	-31.1958	+	j 0.00

Case 4

$T_L = 0.4$, $S = 0.020$

Other machine parameters same as case 1

$$w_b A = \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & -0.72539 & 0.00476 & -0.00326 & 0.68832 \\ 0.00 & -0.01685 & -0.99971 & -0.01295 & 0.01091 \\ 0.00 & 0.68813 & -0.01941 & -0.00768 & 0.72529 \\ 0.00 & 0.00270 & -0.01308 & 0.99988 & 0.00767 \end{bmatrix}$$

Eigen Values

1.	-61.0025	+	j 119.5808
2.	-61.0025	-	j 119.5808
3.	0.0294	+	j 77.4287
4.	0.0294	-	j 77.4287
5.	-30.9778	+	j 0.00

Case 5

Machine Parameters

$r_s = 0.020$ / $r_r' = .015$ / $X_{\ell s} = .1$ / $X_{\ell r'} = .1$ / $X_m = 3.5$

H = .1 /

$V_k = 0.020$ / $V_m = 1.0$

$f_R = 0.27$

$T_L = 0$

$S = 0$

$$V = \begin{bmatrix} 0.290 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$w_b A = \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & -0.71860 & 0.00252 & -0.00228 & 0.69541 \\ 0.00 & -0.01427 & -0.99818 & 0.05763 & -0.01094 \\ 0.00 & 0.69527 & -0.01788 & -0.00118 & 0.71852 \\ 0.00 & 0.00 & 0.05761 & 0.99833 & 0.00307 \end{bmatrix}$$

Eigen Values

1	-51.4648	+	j 116.1080
2	-51.4648	-	j 116.1080
3	0.2141	+	j 78.7702
4	0.2141	-	j 78.7702
5	31.3072	+	j 0.00

Case 6

$T_L = 0.1$, $S = 0.006$

$$w_b A = \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & -0.71942 & 0.00246 & -0.00290 & 0.69457 \\ 0.00 & -0.01429 & -0.99936 & 0.03099 & 0.01112 \\ 0.00 & 0.69443 & -0.01801 & -0.00237 & 0.71933 \\ 0.00 & 0.00 & 0.03095 & 0.99951 & 0.00407 \end{bmatrix}$$

Eigen Values are

1	-51.5461	+	j	115.1083
2	-51.5461	-	j	115.1083
3	0.2698	+	j	78.0538
4	0.2698	-	j	78.0538
5	-31.2560	+	j	0.00

<u>Case 7</u>

$T_L = 0.2$, $S = 0.011$

Other machine parameters as in case 5

$$\begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & -0.72099 & 0.00247 & -0.00302 & 0.69294 \\ 0.00 & -0.01434 & -0.99980 & 0.00817 & 0.01131 \\ 0.00 & 0.69280 & -0.01812 & -0.00370 & 0.72089 \\ 0.00 & 0.00050 & 0.00811 & 0.99996 & 0.00485 \end{bmatrix}$$

<u>Eigen Values</u>

1	-51.5821	+	j	115.0694
2	-51.5821	-	j	115.0694
3	0.2664	+	j	78.0907
4	0.2664	-	j	78.0907
5	-31.1770	+	j	0.00

Case 8

$T_L = 0.3$, $S = 0.017$

Other machine parameters same as case 5.

$$w_b A = \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & -0.72324 & 0.00251 & -0.00294 & 0.69059 \\ 0.00 & -0.01442 & -0.99981 & -0.00604 & 0.01149 \\ 0.00 & 0.69044 & -0.01824 & -0.00499 & 0.72314 \\ 0.00 & 0.00123 & -0.00613 & 0.99996 & 0.00557 \end{bmatrix}$$

Eigen Values

1	-51.5910	+ j	115.8150
2	-51.5910	- j	115.8150
3	0.2386	+ j	78.7352
4	0.2386	- j	78.7352
5	- 31.1036	+ j	0.00

Case 9

$T_L = 0.4$, $S = 0.022$

Other machine parameters same as case 5

$$w_b A = \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & -0.72518 & 0.00253 & -0.00243 & 0.68855 \\ 0.00 & -0.01451 & -0.99969 & -0.01666 & 0.01167 \\ 0.00 & 0.68840 & -0.01834 & -0.00715 & 0.72507 \\ 0.00 & 0.00291 & -0.01679 & -0.99983 & 0.00666 \end{bmatrix}$$

The Eigen Values are

1	-51.6890	+	j 114.0177
2	-51.6890	-	j 114.0177
3	0.2407	+	j 77.4050
4	0.2407	-	j 77.4050
5	-30.9120	+	j 0.00

Case 10

Machine Parameters

$r_s = 0.03$ / $r_r' = 0.015$ / $X_{ls} = 0.1$ / $X_{lr'} = 0.1$ /

$X_m = 3.5$ / $H = 0.1$

$V_k = 0.03$/ $V_m = 1.0$

$f_R = 0.315$

$T_L = 0$

$S = 0$

$$V = \begin{bmatrix} 0.345 \\ 0.00 \\ 0.00 \\ 0.00 \end{bmatrix}$$

$$\begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & -0.71979 & 0.00655 & -0.00300 & 0.69415 \\ 0.00 & -0.01832 & -0.99819 & 0.05657 & 0.00933 \\ 0.00 & 0.69395 & -0.01955 & -0.00162 & 0.71976 \\ 0.00 & 0.00 & 0.05655 & 0.99839 & 0.00378 \end{bmatrix}$$

The Eigen Values are

1	-70.3309	+	j	128.5468
2	-70.3309	-	j	128.5468
3	-0.0172	+	j	79.6175
4	- 0.0172	-	j	79.6175
5	-31.3433	+	j	0.00

Case 11

$S = 0.020 \quad T_L = 0.1$

Other machine parameters

$$w_b A = \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & -0.72081 & 0.00649 & -0.00424 & 0.69309 \\ 0.00 & -0.01861 & -0.99927 & 0.03175 & 0.00980 \\ 0.00 & 0.69288 & -0.02009 & -0.00356 & 0.72076 \\ 0.00 & 0.00 & 0.03170 & 0.99948 & 0.00582 \end{bmatrix}$$

Eigen Values are:

1	-70.4956	+	j	127.8427
2	-70.4956	-	j	127.8427
3	0.4689	+	j	78.8579
4	0.4689	-	j	78.8579
5	-31.9861	+	j	0.00

Case . 12

$S = 0.025, \quad T_L = 0.2$

Other machine parameters as in case 10.

$$w_b A = \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & -0.72218 & 0.00651 & -0.00462 & 0.69166 \\ 0.00 & -0.01867 & -0.99971 & 0.01151 & 0.01001 \\ 0.00 & 0.69145 & -0.02021 & -0.00514 & 0.72212 \\ 0.00 & 0.00073 & 0.01143 & 0.99991 & 0.00702 \end{bmatrix}$$

Eigen Values are:

1	-70.5366	+	j	127.8108
2	-70.5366	-	j	127.8108
3	0.4382	+	j	78.9020
4	0.4382	-	j	78.9020
5	31.8427	+	j	0.00

<u>Case 13</u>

$T_L = 0.3$, $S = 0.030$

Other machine parameters as in case 10

$$\begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & -0.72415 & 0.00656 & -0.00479 & 0.68959 \\ 0.00 & -0.01876 & -0.99977 & -0.00102 & 0.01020 \\ 0.00 & 0.68938 & -0.02032 & -0.00660 & 0.72408 \\ 0.00 & 0.00107 & -0.00112 & 0.99997 & 0.00807 \end{bmatrix}$$

Eigen Values are:

1	-70.5909	+	j	128.4257
2	-70.5909	-	j	128.4257
3	0.4265	+	j	79.6712
4	0.4265	-	j	79.6712
5	-31.7106	+	j	0.00

CHAPTER 4

DYNAMIC ANALYSIS OF MACHINE USING NONLINEAR MODEL

4.1 INTRODUCTION

The transient and steady state performance of the machine in unstable region of operation is investigated in this chapter. It is found that in the unstable region machine speed is subjected to sustained oscillations. The following steps are used for the analysis.

Step 1 : The machine is run at a frequency where it is not unstable i.e. the point lies outside the region of instability (point a (Figure 4.1).

Step 2 : When the machine comes to the steady state at the above frequency, its supply frequency is changed so that the operating point lies in the unstable region of operation. Care is taken to change the supply voltage, as discussed in Chapter 3, according to the equation (3.13). Here it is observed that the machine speed is oscillating.

Step 3 : To make sure that these oscillations are only present in the unstable region, the frequency is switched from the unstable operating point to a frequency outside the unstable region.

In this case, machine speed after the initial transients have died out becomes steady.

All these steps are illustrated by the following diagram.

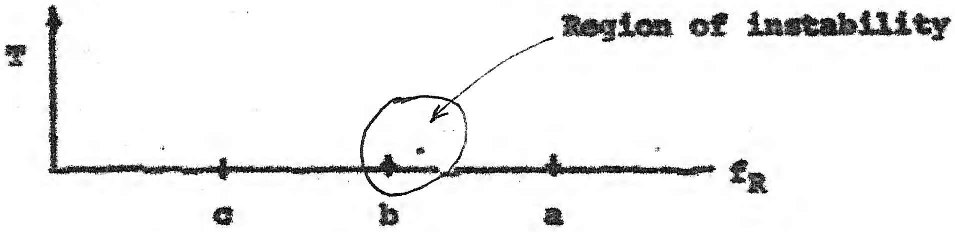


Figure 4.1

Points a, b and c correspond to the operating points in step 1, 2 and 3 respectively. All the above three steps are carried out by simulating the machine on digital computer.

Each step is dealt separately as follows :

Step 1: To find out stable steady state operating point, proceed with the machine model as given in eqns.(2.19) to (2.23).

Equations (2.19), (2.22) and (2.23) are arranged to form the following matrix equation

$$\begin{bmatrix} V_{qs}^{e} \\ V_{ds}^{e} \\ 0 \\ 0 \\ T_L \end{bmatrix} = \begin{bmatrix} r_s+(p/\omega_b)X_s & f_R X_s & (p/\omega_b)X_m & f_R X_m & 0 \\ -f_R X_s & r_s+(p/\omega_b)X_s & -f_R X_m & (p/\omega_b)X_m & 0 \\ (p/\omega_b)X_m & f_R S X_m & r_r'+(p/\omega_b)X_r' & f_R S X_r' & 0 \\ -f_R S X_m & (p/\omega_b)X_m & -f_R S X_r' & r_r'+(p/\omega_b)X_r' & 0 \\ X_m i_{dr}'^{e} & -X_m i_{qr}'^{e} & 0 & 0 & -2Hp \end{bmatrix} \begin{bmatrix} i_{qs}^{e} \\ i_{ds}^{e} \\ i_{qr}'^{e} \\ i_{dr}'^{e} \\ \omega_r/\omega_b \end{bmatrix}$$

where $\quad S = (\omega_e-\omega_r)/\omega_e$ (4.1)

In steady state all the derivatives will have zero value. Thus, in steady state machine is represented by the following nonlinear algebraic equation.

$$\begin{bmatrix} V_{qs}^{e} \\ V_{ds}^{e} \\ 0 \\ 0 \\ T_L \end{bmatrix} = \begin{bmatrix} r_s & f_R X_s & 0 & f_R X_m & 0 \\ -f_R X_s & r_s & -f_R X_m & 0 & 0 \\ 0 & f_R S X_m & r_r' & f_R S X_r' & 0 \\ f_R S X_m & 0 & -f_R S X_r' & r_r' & 0 \\ X_m i_{dr}'^{e} & -X_m i_{qr}'^{e} & 0 & 0 & -2Hp \end{bmatrix} \begin{bmatrix} i_{qs}^{e} \\ i_{ds}^{e} \\ i_{qr}'^{e} \\ i_{dr}'^{e} \\ \omega_r/\omega_b \end{bmatrix}$$

The nonlinear algebraic equations can be solved by the following algorithm by substituting $f_R = f_K a$ as shown in Figure 4.1.

The algorithm is given as follows:

A. PURPOSE

This algorithm finds the minimum of a multivariable, unconstrained, nonlinear function:

Minimize $F(X_1, X_2, \ldots, X_N)$

B. Method :

The procedure is based on the direct search method proposed by H.H. Rosenbrock[3]. No derivatives are required. The procedure assumes a unimodal function; therefore, several sets of starting values for the independent variables should be used if it is known that more than one minimum exists or if the shape of the surface is unknown. The algorithm proceeds as follows :

1) A starting point and initial step sizes, S_i, i = 1,2, ..., N, are picked and the objective function evaluated.

2) The first variable X_1 is stepped a distance S_1 parallel to the axis, and the function evaluated. If the value of F decreased, the move is termed a success and S_1 increased by a factor α,

$\alpha \geq 1.0$. If the value of F increased the move is termed failure and S_1 decreased by a factor β, $0 < \beta \leq 1.0$, and the direction of movement **reversed**.

3) The next variable, X_i, is in turn stepped a distance S_i parallel to the axis. The same acceleration or deceleration and reversal procedure is followed for all variables in consecutive repetitive sequences until a success (decrease in F) and failure (increase in F) have been encountered in all N directions.

4) The axes are then rotated by the following equations. Each rotation of the axes is termed a stage.

$$M_{i,j}^{(k+1)} = \frac{D_{i,j}^{(k)}}{\left[\sum_{\ell=1}^{N} \left(D_{\ell,j}^{(k)}\right)^2\right]^{\frac{1}{2}}}$$

where

$$D_{i,1}^{(k)} = A_{i,1}^{(k)}$$

$$D_{i,j}^{(k)} = A_{i,j}^{(k)} - \sum_{\ell=1}^{j-1}\left[\left(\sum_{n=1}^{j} M_{n,\ell}^{(k)}\right) \cdot A_{n,j}^{(k)}\right] \cdot M_{i,}^{(k+1)} \qquad ,j=2,3,\ldots,N$$

$$A_{i,j}^{(k)} = \sum_{\ell=j}^{N} d_\ell^{(k)} \cdot M_{i,\ell}^{(k)}$$

where

i = variable index = 1,2,3,....,N

j = direction index = 1,2,3,...,N

k = stage index

d_i = sum of distances moved in the i direction since last rotation of axes

$M_{i,j}$ = direction vector component (normalized).

5) Search is made in each of the X-directions using the new coordinate axes

$$\text{new } x_i^{(k)} = \text{old } x_i^{(k)} + x_j^{(k)} \, M_{i,j}^{(k)}$$

6) The procedure terminates when the convergence criterion is satisfied.

(For programme description see reference 4).

The solution vector $[i_{qso}{}^{e}, i_{dso}{}^{e}, i_{qro}{}'^{e}, i_{dro}{}'^{e}, \omega_r/\omega_b]^T$ is the steady state operation point.

Step 2 : Writing (4.1) concisely in partitioned matrix notation as

$$\begin{bmatrix} \underline{V} \\ \\ T_L \end{bmatrix} = \begin{bmatrix} \underline{R} & \underline{0} \\ \\ \underline{V}_{20}^{T} & 0 \end{bmatrix} \begin{bmatrix} \underline{i} \\ \\ \omega_r/\omega_b \end{bmatrix} + p/\omega_b \begin{bmatrix} \underline{X} & 0 \\ \\ \underline{0} & -2H\omega_b \end{bmatrix} \begin{bmatrix} \underline{i} \\ \\ \omega_r/\omega_b \end{bmatrix} \tag{4.3}$$

$$\underline{V} = \begin{bmatrix} v_{qs}{}^{e} \\ v_{ds}{}^{e} \\ 0 \\ 0 \end{bmatrix} \tag{4.4}$$

$$\underline{i} = \begin{bmatrix} i_{qs}{}^{e} \\ i_{ds}{}^{e} \\ i_{qr}{}'^{e} \\ i_{dr}{}'^{e} \end{bmatrix} \tag{4.5}$$

$$\underline{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{4.6}$$

$$\underline{V}_{20} = \begin{bmatrix} X_m i_{dr}'^e \\ -X_m i_{qr}'^e \\ 0 \\ 0 \end{bmatrix} \tag{4.7}$$

$$\underline{R} = \begin{bmatrix} r_s & f_R X_s & 0 & f_R X_m \\ -f_R X_s & r_s & -f_R X_m & 0 \\ 0 & f_R S_o X_m & r_r' & f_R S_o X_r' \\ -f_R S_o X_m & 0 & -f_R S_o X_r' & r_r' \end{bmatrix} \tag{4.8}$$

$$\underline{X} = \begin{bmatrix} X_s & 0 & X_m & 0 \\ 0 & X_s & 0 & X_m \\ X_m & 0 & X_r' & 0 \\ 0 & X_m & 0 & X_r' \end{bmatrix} \tag{4.9}$$

Upon solving for the vector denoting the time derivatives of currents and speed in (4.3)we obtain

$$p/\omega_b \begin{bmatrix} i \\ \\ \omega_r/\omega_b \end{bmatrix} = \begin{bmatrix} -X^{-1}R & \underline{0} \\ \\ (1/2H\omega_b)V_{20}^T & 0 \end{bmatrix} \begin{bmatrix} i \\ \\ \omega_r/\omega_b \end{bmatrix} + \begin{bmatrix} X^{-1} & 0 \\ \\ 0 & -(1/2H\omega_b) \end{bmatrix} \begin{bmatrix} V \\ \\ T_L \end{bmatrix}$$

(4.10)

Equation (4.10) constitutes the vector matrix nonliner differential equation describing the operation of the machine.

In equation (4.10) f_{Rb} as shown in diag. (4.1) for step 2 (4.10) is solled by Rk fourth order method on the digital computer. Solution vector found in step 1 is used for initial conditions to this step.

Step 3 : When the machine speed , ω_r/ω_b exhibits steady state oscillations , (when oscillation become periodic) , f_{Rb} is changed to f_{Rs} in equation (4.10) to make the supply frequency lie outside the unstable region as shown in Figure 4.1.

The equation is again solved on similar lines as discussed in step 2. After transients die out, the machine comes to a new steady state point as in step 1.

4.2 TRANSFORMATION OF STATOR VOLTAGES

The machine is supplied with 3 phase voltages - V_{as}, V_{bs} and V_{cs}.

But in the above equation (4.2) (Q_s) quadrature and (D_s) direct axis components of these voltages are used. In order to find out these components from the known phase voltages the following steps are used.

Step 1 : In the first step the Q_s and D_s applied voltages, in a reference frame fixed in stator, are obtained by setting $\theta = 0$ in equation (2.7).

Thus,

$$V_{qs}^{s} = 2/3\ (V_{as}-(1/2)V_{bs} - (1/2)V_{cs}) \qquad (4.11)$$

$$V_{ds}^{s} = (1/\sqrt{3})(-V_{bs} + V_{cs}) \qquad (4.12)$$

The raised index s is used to denote variables in reference frame fixed in the stator.

Step 2: After step 1, it becomes convenient to express the applied voltages in a synchronously rotating reference frame as functions of V_{qs}^{s} and V_{ds}^{s}. Thus,

$$V_{qs}^{e} = V_{qs}^{s} \cos \theta e - V_{ds}^{s} \sin \theta e \qquad (4.13)$$

$$V_{ds}^{e} = V_{qs}^{s} \sin \theta e + V_{ds}^{s} \cos \theta e \qquad (4.14)$$

The raised index e is used to denote variables in synchronously rotating reference frame. Now in this case the per unit line voltage which is also equal to per unit phase voltage is known.

Thus, on substituting the values V_{as}, V_{bs}, V_{cs} in equations (4.11) to (4.14) we obtain

V_{qs}^{e} = Amplitude of phase voltage

$V_{ds}^{e} = 0$

The voltage vector in equation (4.10) can be written in the form

$$V = \begin{bmatrix} \text{P.U. phase voltage amplitude} \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (4.15)$$

This study was carried out for the machine whose specifications are given as follows.: 7.5 HP/220V/60Hz)4 pole/3-phase.

In variable speed systems, the amplitude of the applied voltage is decreased as frequency decreases in order to avoid saturation of the machine. However, if the voltage is decreased as a linear function of frequency, the breakdown torque is depleted significantly at low frequencies since an increased percentage of the applied voltage is dropped across the stator resistance as frequency is reduced. In this study, as a simple

means of i_R compensation the voltage required to produce rated flux linkage at rated load (T_L = 1.0 p.u.) and rated speed (f_R = 1.0) has been predetermined from the steady state considerations. For the set (1) of parameters the terminal voltage required to satisfy this constraint is V = 1.025 p.u. When operating from a variable frequency source, the terminal voltage has been adjusted so that for any frequency

$$V = V_k + f_R V_m \qquad (4.16)$$

where V_k = 0.025 and V_m = 1.0 p.u. From the form of (4.16) it is clear that the constant factor V_k serves to compensate for the stator i_R drop.

It can be noted that when f_R = 1.0, the terminal voltage V = 1.025.

Thus, for different operating conditions, the proper input voltages are used. The stability boundaries for this motor are given in Figures 3.1 to 3.3 for different machine parameters and load torques. The amplitude and frequency of speed fluctuations in the unstable region, are computed as given in Section 4.2., and tabulated below.

4.3 Results :

Set 1: Machine parameters:

$r_s = .025$

$r_r' = .015$

$X_{\ell s} = .1$

$X_{\ell r}' = .1$

$X_m = 3.5$

$H = .1$

$V_k = .025$

$V_m = 1.0$

$f_R = 0.4$

For the above case $V_{qs} = .025 + .4 = .425$ and the voltage vector

$$= \begin{bmatrix} .425 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The following initial steady state operating points are considered

$f_{Ra} = .4, \; V_{qs} = .425$

$f_{Rb} = .3, \; V_{qs} = .025 + .3 = .325$

$f_{Rc} = .25, V_{qs} = .025 + .25 = .275$

The amplitude and frequency of the oscillation input observed with different values of torque iwhen the input frequency

is changed to $f_{Rb} = .3$, which corresponds to the unstable operating point are shown below.

Torque P.U.	Lower crest of speed	Upper crest of speed	amplitude of oscilla-tions	frequency of osci-llations	Table No.
0	.2544	.3410	.0408	12.82	1
.1	.2582	.3394	.0406	12.82	2
.2	.2581	.3367	.0393	12.82	3
.3	.2591	.3328	.0368	12.66	4
.4	.2614	.3274	.0340	12.66	5

Set II : Machine parameters :

r_s = $\underline{.030}$ (changed)

r_r' = .015

$X_{\ell s}$ = .1

$X_{\ell r}'$ = .1

X_m = 3.5

H = .1

V_R = .030

V_m = 1.0

f_R = .4

For the above case V_{qs} = .03 + .40 = .430 and the voltage vector

$$= \begin{bmatrix} .380 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The following initial steady state operating points are considered

f_{Ra} = .4, V_{qs} = .03 + .40 = .430

f_{Rb} = .35, V_{qs} = .03 + .35 = .380

f_{Rc} = .3, V_{qs} = .03 + .3 = .33

The amplitude and frequency of the oscillation observed with different values of torque when the input frequency is changed to f_{Rb} = .35, which corresponds to the unstable operating point are shown below:

Torque P.U.	Lower crest of speed	Upper crest of speed	amplitude of oscillations	frequency of oscillations	Table No.
.1	.3163	.3816	.0326	13.33	6
.2	.3201	.3747	.0273	13.33	7
.3	.3254	.3664	.0206	13.33	8
.4	.3314	.3572	.0129	13.33	9

Set III : Machine parameters :

r_s = <u>.020</u> (changed)

r_r' = .015

$X_{\ell s}$ = .1

$X_{\ell r}'$ = .1

X_m = 3.5

H = .1

V_k = .020

V_m = 1.0

f_R = 0.4

For the above case V_{qs} = .020 + .4 = .420 and the voltage vector

$$\begin{bmatrix} .320 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The following intitial steady state operating points are considered

f_{Ra} = .4, V_{qs} = .020 + .4 = .420

f_{Rb} = .3, V_{qs} = .020 + .3 = .320

f_{Rc} = .2, V_{qs} = .020 + .2 = .220

The amplitude and frequency of the oscillation observed with different values of torque when the input frequency is changed to f_{Rb} = .3, which corresponds to the unstable operating point are shown below :

Torque p.u.	Lower crest of speed	Upper crest of speed	amplitude of oscillations	frequency of oscillations	Table No.
.0	.2342	.3667	.0662	13.33	10
.1	.2345	.3635	.0645	13.96	11
.2	.2356	.3594	.0619	13.16	12
.3	.2375	.3544	.0584	13.00	13
.4	.2404	.3483	.0539	13.00	14

Set IV Machine parameters:

r_s = .025

r_r' = .010 (changed)

$X_{\ell s}$ = .1

$X_{\ell r}'$ = .1

X_m = .35

H = .1

V_k = .025

V_m = 1.0

f_R = 0.5

For the above case V_{qs} = .025 + .5 = .525 and the voltage vector

$$\begin{bmatrix} .425 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The following initial steady state operating points are considered

f_{Ra} = .5, V_{qs} = .025 + .5 = .525

f_{Rb} = .4, V_{qs} = .025 + .4 = .425

f_{Rc} = .2, V_{qs} = .025 + .2 = .225

The amplitude and frequency of the oscillation observed with different values of torque when the input frequency is changed to f_{Rb} = .4, which corresponds to the unstable operating point are shown below :

Torque p.u.	Lower crest of speed	Upper crest of speed	amplitude of oscillations	frequency of oscillations	Table No.
0	.3073	.4970	.0948	14.09	15
.1	.3099	.4921	.0911	14.09	16
.3	.3171	.4802	.0815	14.09	17

Set V: Machine parameters:

r_s = .025

r_r' = .015

$X_{\ell r}'$ = .1

X_m = 3.5

H = .05 (changed)

V_k = .025

V_m = 1.0

f_R = .5

For the above case V_{qs} = .025 + .5 = .525 and the voltage vector

$$\begin{bmatrix} .425 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The following initial steady state operating points are considered

f_{Ra} = .5, V_{qs} = .025 + .5 = .525

f_{Rb} = .4, V_{qs} = .025 + .4 = .425

f_{RC} = .25, V_{qs} = .025 + .25= .275

The amplitude and frequency of the oscillation observed with different values of torque when the input frequency is changed to $f_{Rb} = .4$, which corresponds to the unstable operating point are shown below :

Torque p.u.	Lower crest of speed	Upper crest of speed	amplitude of oscillations	frequency of oscillations	Table No.
0	.2253	.5716	.1731	18.18	18
.1	.2263	.5684	.1710	18.18	19
.25	.2287	.5619	.1666	17.86	20
.4	.2328	.5534	.1603	17.86	21

Set VI

In this set, load torque is not changed. With a constant load torque the oscillations are observed for different frequencies within the unstable region.

Machine parameters :

r_s = .025

r_r' = .015

$x_{\ell s}$ = .1

$x_{\ell r}'$ = .1

X_m = 3.5

H = .1

V_k = .025

V_m = 1.0

The following initial study state operating points are considered :

$f_{Ra} = .4$, $V_{qs} = .025 + .4 = .425$

$f_{Rc} = .25$, $V_{qs} = .025 + .25 = .275$

$f_{Rb1} = .3$, $V_{qs} = .025 + .3 = .325$

$f_{Rb2} = .31$, $V_{qs} = .025 + .31 = .335$

$f_{Rb3} = .33$, $V_{qs} = .025 + .33 = .355$

$f_{Rb4} = .35$, $V_{qs} = .025 + .35 = .375$

$f_{Rb5} = .36$, $V_{qs} = .025 + .36 = .385$

$T_L = 0$, for all the cases.

CHAPTER 5

OSCILLATION FREQUENCY FROM EIGEN VALUES

5.1 INTRODUCTION

In this chapter oscillation frequency is found from the eigen values. When the system is critically stable the oscillation frequency can be found exactly by the complex eigen values having real parts equal to zero. Mathematically, it is shown below.

For this case, eigenvalues are given by these values of for which the determinant

$$\left| \lambda \begin{bmatrix} I & 0 \\ 0^T & 1 \end{bmatrix} + \omega_b \begin{bmatrix} X^{-1}R & X^{-1}V_{10} \\ -(1/2H\omega_b)V_{20}^T & 0 \end{bmatrix} \right| = 0$$

It is found that these eigen values are of the form

$$\lambda_1 \; , \lambda_2 = \sigma_1 \pm j\omega_1$$

$$\lambda_3 \; , \lambda_4 = \sigma_2 \pm j\omega_2$$

$$\lambda_5 \quad\quad = \sigma_3$$

They are two pairs of conjugate complex values and one real value. Now when the system is critically stable, one conjugate complex pair has real part $\sigma_1 = 0$.

Thus, for critical stable system the fives eigen values are as follows ;

$0 \pm j\,\omega_1$

$\sigma_2 + j\,\omega_2$

σ_3

σ_2 and σ_3 are negative.

The output will be of the form

$$A_1\, e^{\sigma_1 t} \sin(\omega_1 t + \phi_1) + A_2\, e^{\sigma_2 t} \sin(\omega_2 t + \phi_2) + A_3\, e^{\sigma_3 t}.$$

For critically stable case, $\sigma_1 = 0$ and σ_2, σ_3 are negative. Thus the second and third term which correspond to negative real parts of eigen values decrease exponentially with time. First term will be periodic sinusoid as $\sigma_1 = 0$. So in the steady state, the output is given by

$$\text{Output} = A_1 \sin(\sigma_1 t + \phi_1)$$

Hence, it can be established that in the critical stable case the system oscillates with frequency ω_1.

5.2 ANALYSIS USING LINEAR MODEL

The oscillation frequency found from linearized model is the same as that found from nonlinear model if the perturbation are small.

Next, it is found that oscillation frequencies for different f_{Rb}'s in the region of instability do not differ much. One such set is given in set 6 in Chapter 4. Here the machine is run at different operating points lying in the region of instability. It is found that with the change in supply frequency within the boundary of region of instability, the oscillation frequency does not change appreciably.

Thus, oscillation frequency for any operating point in the region of instability is approximately equal to ω_1 (which can be found from eigen values as discussed in thebbeginning).

The results obtained by this method are compared to those found by simulating nonlinear model and are given in tabular form as follows:

Machine parameters, and another informatian which is not given with each set can be referred from the same set number in Chapter.4.

5.3 Results :

Here in each set, four quantities are tabulated they are load torque, eigen value which gives the oscillation frequency found by above mentioned method and oscillation frequency as found in Chapter 4.

Set I (Ref Set I, Chap 4)

Load Torque	Pair of conjugate complex eigen values with real part almost equal to zero: Real part σ_2	Imaginary part , $\pm w_2$	oscillation frequency from eigen value = Imaginary part/2π	oscillation frequency as found in Chapter 2
0	0.0901	$\pm$ 78.02	12.41	12.82
.1	0.0929	$\pm$ 78.1368	12.42	12.82
.2	0.0966	$\pm$ 78.1800	12.42	12.82
.3	0.0812	$\pm$ 78.8939	12.52	12.66
.4	0.0294	$\pm$ 77.4287	12.32	12.66
Set II (Ref Set II, Chapter 4)				
.1	0.4689	$\pm$ 78.8579	12.52	13.33
.2	0.4382	$\pm$ 78.9020	12.52	13.33
.3	0.4265	$\pm$ 79.6712	12.65	13.33
.4	0.2824	$\pm$ 78.0949	12.42	13.33
Set III (Ref. Set III, Chapter 4)				
.0	0.2141	$\pm$ 78.7702	12.52	13.33
.1	0.2698	$\pm$ 78.0538	12.42	13.16
.2	0.2664	$\pm$ 78.0907	12.42	13.16
.3	0.2386	$\pm$ 78.7352	12.52	13.00
.4	0.2407	$\pm$ 77.4050	12.32	13.00

Load Torque	Pair of congugate complex eigen values with real part almost equal to zero: Real part, σ_2	Pair of congugate complex eigen values with real part almost equal to zero: Imaginary part, $\pm w_2$	oscillation frequency from eigen value = $\frac{\text{Imaginary part}}{2\pi}$	oscillation frequency as found in Chapter IV
Set IV (Ref. Set IV, Chapter 4)				
0	0.2231	$\pm$ 82.6578	13.3	14.09
.1	0.3454	$\pm$ 82.2100	13.3	14.09
.3	0.1230	$\pm$ 84.9002	13.5	14.09
Set V (Ref. Set V, Chapter 5)				
0	0.2732	108.2168	17.2	18.18
.1	0.1811	109.1807	17.3	18.18
.25	0.1935	108.9210	17.3	17.86
.4	0.2654	108.2096	17.2	17.86

From the above tables, it is found by comparison that frequency computed by this method is almost equal to that found by simulating the non-linear model for any operating point in the region of instability.

Table No.1

(In this table, only steady state speed oscillations are tabulated. Time = 0, when steady state is reached).

Time	W_r/W_b	Time	W_r/W_b
0.001	0.298688	0.033	0.315545
0.002	0.301634	0.037	0.312167
0.003	0.304576	0.038	0.308695
0.004	0.307503	0.039	0.305154
0.005	0.310400	0.040	0.301576
0.006	0.313254	0.041	0.297988
0.007	0.316051	0.042	0.294419
0.008	0.318778	0.043	0.290900
0.009	0.321420	0.044	0.287457
0.010	0.323962	0.045	0.284118
0.011	0.326389	0.046	0.280910
0.012	0.328685	0.047	0.277857
0.013	0.330835	0.048	0.274982
0.014	0.332822	0.049	0.272306
0.015	0.334631	0.050	0.269848
0.016	0.336247	0.051	0.267623
0.017	0.337654	0.052	0.265646
0.018	0.338839	0.053	0.263928
0.019	0.339787	0.054	0.262478
0.020	0.340486	0.055	0.261301
0.021	0.340927	0.056	0.260402
0.022 (Higher	0.341099	0.057	0.259781
0.023 Crest).	0.340994	0.058	0.259437
0.024	0.340609	0.059 (Lower	0.259365
0.025	0.339939	0.060 Crest)	0.259562
0.026	0.338985	0.061	0.260018
0.027	0.337748	0.062	0.260724
0.028	0.336234	0.063	0.261171
0.029	0.334448	0.064	0.262846
0.030	0.332402	0.065	0.264237
0.031	0.330108	0.066	0.265829
0.032	0.327583	0.067	0.267609
0.033	0.324842	0.068	0.269563
0.034	0.321907	0.069	0.271675
0.035	0.318800	0.070	0.273932

Time	W_r/W_b	Time	W_r/W_b
0.071	0.276319	0.110	0.327803
0.072	0.278821	0.111	0.325080
0.073	0.281426	0.112	0.322162
0.074	0.281184	0.113	0.319069
0.075	0.286884	0.114	0.315826
0.076	0.289712	0.115	0.312459
0.077	0.292589	0.116	0.308994
0.078	0.295501	0.117	0.305459
0.079	0.298436	0.118	0.301883
0.080	0.301381	0.119	0.298296
0.081	0.304324	0.120	0.294725
0.082	0.307251	0.121	0.291201
0.083	0.310151	0.122	0.287751
0.084	0.313009	0.123	0.284403
0.085	0.315811	0.124	0.281184
0.086	0.318544	0.125	0.278117
0.087	0.321194	0.126	0.275227
0.088	0.323744	0.127	0.272523
0.089	0.326181	0.128	0.270056
0.090	0.328489	0.129	0.267881
0.091	0.330651	0.130	0.265812
0.092	0.332652	0.131	0.264072
0.093	0.334477	0.132	0.262599
0.094	0.336110	0.133	0.261399
0.095	0.337535	0.134	0.260476
0.096	0.338739	0.135	0.259831
0.097	0.339708	0.136	0.259463
0.098	0.340429	0.137 (Lower	0.259368
0.099	0.340892	0.138 Crest)	0.259542
0.100 (Higher	0.341087	0.139	0.259976
0.101 Crest)	0.341006	0.140	0.260661
0.102	0.340645	0.141	0.261588
0.103	0.340000	0.142	0.262743
0.104	0.339071	0.143	0.264115
0.105	0.337858	0.144	0.265690
0.106	0.336367	0.145	0.267455
0.107	0.334605	0.146	0.269394
0.108	0.332581	0.147	0.271493
0.109	0.330309	0.148	0.273737

Table No. 2

TIME	ω_r/ω_b	TIME	ω_r/ω_b
0.1	0.296746	4.6	0.267434
0.2	0.240705	4.7	0.325606
0.3	0.313211	4.8	0.324677
0.4	0.353368	4.9	0.260597
0.5	0.264940	5.0	0.287181
0.6	0.262022	5.1	0.338639
0.7	0.335685	5.2	0.298317
0.8	0.328636	5.3	0.259231
0.9	0.250010	5.4	0.310129
1.0	0.287330	5.5	0.336631
1.1	0.347198	5.6	0.272492
1.2	0.296100	5.7	0.272233
1.3	0.253043	5.8	0.329946
1.4	0.312789	5.9	0.318191
1.5	0.342199	6.0	0.258896
1.6	0.268017	6.1	0.293298
1.7	0.269466	6.2	0.339495
1.8	0.333599	6.3	0.290823
1.9	0.320527	6.4	0.261759
2.0	0.254671	6.5	0.315958
2.1	0.292567	6.6	0.332934
2.2	0.343197	6.7	0.267449
2.3	0.290729	6.8	0.277498
2.4	0.258661	6.9	0.333692
2.5	0.316416	7.0	0.311113
2.6	0.336024	7.099	0.258250
2.7	0.265994		(Lower Crest)
2.8	0.275450	7.100	0.258282
2.9	0.33499	7.101	0.258574
3.0	0.313205	7.102	0.259118
3.1	0.256244	7.103	0.259904
3.2	0.298270	7.104	0.260922
3.3	0.341263	7.105	0.262160
3.4	0.284520	7.106	0.263605
3.5	0.263032	7.107	0.265243
3.6	0.320959	7.108	0.267060
3.7	0.330533	7.109	0.269044
3.8	0.263129	7.110	0.271178
3.9	0.281244	7.111	0.273450
4.0	0.336984	7.112	0.275844
4.1	0.305832	7.113	0.278348
4.2	0.257485	7.114	0.280948
4.3	0.304178	7.115	0.283631
4.4	0.339315	7.116	0.286383
4.5	0.278291	7.117	0.289192

TIME	ω_r/ω_b	TIME	ω_r/ω_b
7.118	0.292046	7.163	0.286068
7.119	0.294932	7.164	0.282764
7.120	0.297837	7.165	0.279591
7.121	0.300750	7.166	0.276571
7.122	0.303658	7.167	0.273728
7.123	0.306548	7.168	0.271082
7.124	0.309408	7.169	0.268650
7.125	0.312224	7.170	0.266449
7.126	0.314983	7.171	0.264493
7.127	0.317671	7.172	0.262792
7.128	0.320273	7.173	0.261355
7.129	0.322775	7.174	0.260188
7.130	0.325162	7.175	0.259294
7.131	0.327418	7.176	0.258673
7.132	0.329528	7.177	0.258325
7.133	0.331476	7.178	0.258246 (Lower Crest)
7.134	0.333247	7.179	0.258429
7.135	0.334825	7.180	0.258869
7.136	0.336196	7.181	0.259555
7.137	0.337345	7.182	0.260478
7.138	0.338260	7.183	0.261625
7.139	0.338927	7.184	0.262984
7.140	0.339338	7.185	0.264543
7.141	0.339483 (Higher Crest)	7.186	0.266287
7.142	0.339354	7.187	0.268202
7.143	0.338947	7.188	0.270274
7.144	0.338259	7.189	0.272490
7.145	0.337289	7.190	0.274834
7.146	0.336041	7.191	0.277294
7.147	0.334518	7.192	0.279855
7.148	0.332728	7.193	0.282504
7.149	0.330682	7.194	0.285228
7.150	0.328392	7.195	0.288014
7.151	0.325873	7.196	0.290850
7.152	0.323144	7.197	0.293724
7.153	0.320224	7.198	0.296622
7.154	0.317136	7.199	0.299532
7.155	0.313902	7.200	0.302443
7.156	0.310550	7.201	0.305342
7.157	0.307104	7.202	0.308225
7.158	0.303594	7.203	0.311051
7.159	0.300047	7.204	0.313835
7.160	0.296493	7.205	0.316553
7.161	0.292959	7.206	0.319193
7.162	0.289475	7.207	0.321738

TIME	ω_r/ω_b	TIME	ω_r/ω_b
7.208	0.324174	7.251	0.261932
7.209	0.326486	7.252	0.260651
7.210	0.328659	7.253	0.259643
7.211	0.330676	7.254	0.258908
7.212	0.332522	7.255	0.258446
7.213	0.334182	7.256	0.258255 (Lower Crest)
7.214	0.335641		
7.215	0.336884		
7.216	0.337898		
7.217	0.338670		
7.218	0.339189		
7.219	0.339446 (Higher Crest)		
7.220	0.339432		
7.221	0.339142		
7.222	0.338571		
7.223	0.337719		
7.224	0.336587		
7.225	0.335179		
7.226	0.333500		
7.227	0.331560		
7.228	0.329370		
7.229	0.326946		
7.230	0.324303		
7.231	0.321461		
7.232	0.318441		
7.233	0.315266		
7.234	0.311960		
7.235	0.308551		
7.236	0.305066		
7.237	0.301531		
7.238	0.297977		
7.239	0.294432		
7.240	0.290925		
7.241	0.287483		
7.242	0.284134		
7.243	0.280904		
7.244	0.277818		
7.245	0.274899		
7.246	0.272169		
7.247	0.269646		
7.248	0.267347		
7.249	0.265288		
7.250	0.263480		

TABLE No. 3

Time	ω_r/ω_b	Time	ω_r/ω_b
.1	0.297735	4.0	0.289556
.2	0.238453	4.1	0.337194
.3	0.306092	4.2	0.295472
.4	0.353644	4.3	0.258459
.5	0.273448	4.4	0.306999
.6	0.253803	4.5	0.535623
.7	0.323665	4.6	0.275518
.8	0.338791	4.7	0.267363
.9	0.257351	4.8	0.322821
1.0	0.271206	4.9	0.324190
1.1	0.336890	5.0	0.261935
1.2	0.317686	5.1	0.281804
1.3	0.250939	5.2	0.333877
1.4	0.289965	5.3	0.305418
1.5	0.343029	5.4	0.257728
1.6	0.293963	5.5	0.298822
1.7	0.254119	5.6	0.336969
1.8	0.308544	5.7	0.284541
1.9	0.339697	5.8	0.262802
2.0	0.272909	5.9	0.315638
2.1	0.264916	6.0	0.330199
2.2	0.324845	6.1	0.267540
2.3	0.326633	6.2	0.274864
2.4	0.259265	6.3	0.329263
2.5	0.280523	6.4	0.314503
2.6	0.336051	6.5	0.258805
2.7	0.306580	6.6	0.290915
2.8	0.255394	6.7	0.336385
2.9	0.298229	6.8	0.294001
3.0	0.339128	6.9	0.259714
3.1	0.284791	7.0	0.308090
3.2	0.260867	7.099	0.334276
3.3	0.315512	7.100	0.332987
3.4	0.332250	7.101	0.331437
3.5	0.267138	7.102	0.329635
3.6	0.273249	7.103	0.327590
3.7	0.329572	7.104	0.325315
3.8	0.316345	7.105	0.322827
3.9	0.257913	7.106	0.320142

Time	ω_r/ω_b	Time	ω_r/ω_b
7.107	0.317280	7.148	0.284398
7.108	0.314264	7.149	0.287090
7.109	0.311115	7.150	0.289829
7.110	0.307860	7.151	0.292604
7.111	0.304522	7.152	0.295403
7.112	0.301130	7.153	0.298214
7.113	0.297710	7.154	0.301025
7.114	0.294290	7.155	0.303823
7.115	0.290897	7.156	0.306598
7.116	0.287559	7.157	0.309335
7.117	0.284300	7.158	0.312022
7.118	0.281147	7.159	0.314646
7.119	0.278124	7.160	0.317192
7.120	0.275253	7.161	0.319648
7.121	0.272555	7.162	0.321997
7.122	0.270050	7.163	0.324227
7.123	0.267753	7.164	0.326322
7.124	0.265680	7.165	0.328266
7.125	0.263844	7.166	0.330045
7.126	0.262253	7.167	0.331645
7.127	0.260916	7.168	0.333051
7.128	0.259837	7.169	0.334249
7.129	0.259019	7.170	0.335226
7.130	0.258464	7.171	0.335971
7.131	0.258168	7.172	0.336472
7.132	0.258128	7.173	0.336722
	(Lower Crest)		(Higher Crest)
7.133	0.258340	7.174	0.336711
7.134	0.258794	7.175	0.336436
7.135	0.259484	7.176	0.335891
7.136	0.260398	7.177	0.335077
7.137	0.261526	7.178	0.333993
7.138	0.262856	7.179	0.332644
7.139	0.264375	7.180	0.331036
7.140	0.266071	7.181	0.329178
7.141	0.267930	7.182	0.327079
7.142	0.269939	7.183	0.324755
7.143	0.272084	7.184	0.322221
7.144	0.274352	7.185	0.319494
7.145	0.276731	7.186	0.316596
7.146	0.279216	7.187	0.313548
7.147	0.281766	7.188	0.310373

Time	ω_r/ω_b	Time	ω_r/ω_b
7.189	0.307097	7.229	0.290465
7.190	0.303746	7.230	0.293246
7.191	0.300346	7.231	0.296048
7.192	0.296925	7.232	0.298859
7.193	0.293510	7.233	0.301667
7.194	0.290128	7.234	0.304461
7.195	0.286806	7.235	0.307227
7.196	0.283571	7.236	0.309953
7.197	0.280446	7.237	0.312626
7.198	0.277457	7.238	0.315233
7.199	0.274624	7.239	0.317759
7.200	0.271970	7.240	0.320191
7.201	0.249512	7.241	0.322514
7.202	0.267266	7.242	0.324713
7.203	0.265247	7.243	0.326774
7.204	0.263466	7.244	0.328681
7.205	0.261933	7.245	0.330420
7.206	0.260655	7.246	0.331976
7.207	0.259637	7.247	0.333335
7.208	0.258879	7.248	0.334483
7.209	0.258383	7.249	0.335408
7.210	0.258147	7.250	0.336097
	(Lower crest)	7.251	0.336542
7.211	0.258165	7.252	0.336732
7.212	0.258433		(Higher crest)
7.213	0.258942	7.253	0.336661
7.214	0.259684		
7.215	0.260648		
7.216	0.261823		
7.217	0.263197		
7.218	0.264757		
7.219	0.266491		
7.220	0.268385		
7.221	0.270426		
7.222	0.272600		
7.223	0.274894		
7.224	0.277295		
7.225	0.279791		
7.226	0.282368		
7.227	0.285014		
7.228	0.287717		

TABLE No. 4

TIME	ω_r/ω_b	TIME	ω_r/ω_b
0.1	0.298967	4.5	0.303421
0.2	0.236535	4.6	0.332813
0.3	0.298468	4.7	0.279655
0.4	0.352097	4.8	0.264121
0.5	0.283774	4.9	0.314200
0.6	0.247418	5.0	0.327267
0.7	0.309783	5.1	0.268837
0.8	0.344044	5.2	0.272084
0.9	0.271106	5.3	0.323393
1.0	0.257633	5.4	0.317709
1.1	0.320087	5.5	0.261677
1.2	0.334014	5.6	0.282035
1.3	0.261548	5.7	0.329975
1.4	0.268470	5.8	0.305290
1.5	0.328648	5.9	0.258818
1.6	0.321552	6.0	0.293078
1.7	0.255783	6.1	0.332964
1.8	0.280006	6.2	0.291687
1.9	0.334593	6.3	0.260237
2.0	0.307315	6.4	0.304348
2.1	0.254124	6.5	0.331651
2.2	0.291916	6.6	0.278791
2.3	0.337050	6.7	0.265390
2.4	0.292596	6.8	0.314946
2.5	0.256428	6.9	0.325857
2.6	0.303686	7.0	0.268340
2.7	0.335362	7.099	0.273432
2.8	0.278974	7.100	0.275621
2.9	0.262162	7.101	0.277905
3.0	0.314651	7.102	0.280275
3.1	0.329338	7.103	0.282711
3.2	0.267961	7.104	0.285221
3.3	0.270559	7.105	0.287775
3.4	0.323977	7.106	0.290368
3.5	0.319424	7.107	0.292988
3.6	0.260696	7.108	0.295624
3.7	0.280779	7.109	0.298265
3.8	0.330706	7.110	0.300899
3.9	0.306725	7.111	0.303514
4.0	0.257745	7.112	0.306099
4.1	0.292000	7.113	0.308642
4.2	0.333888	7.114	0.311129
4.3	0.292856	7.115	0.313549
4.4	0.259066	7.116	0.315887

Time	ω_r/ω_b	Time	ω_r/ω_b
7.117	0.318132	7.211	0.331737
7.118	0.320268	7.212	0.330877
7.119	0.322283	7.213	0.329769
7.120	0.324162	7.214	0.328418
7.121	0.325891	7.215	0.326830
7.122	0.327458	7.240	0.262413
7.123	0.328848	7.241	0.261267
7.124	0.330050	7.242	0.260360
7.125	0.331050	7.243	0.259695
7.126	0.331838	7.244	0.259271
7.127	0.332404	7.245	0.259087
7.128	0.332738		(Lower crest)
7.129	0.332834	7.246	0.259137
	(Higher crest)		
7.130	0.332686		
7.131	0.332289		
7.132	0.331643		
7.133	0.330746		
7.134	0.329602		
7.160	0.263568		
7.161	0.262219		
7.162	0.261107		
7.163	0.260235		
7.164	0.259605		
7.165	0.259216		
7.166	0.259065		
	(Lower crest)		
7.167	0.259150		
7.168	0.259463		
7.169	0.259997		
7.170	0.260745		
7.171	0.261697		
7.172	0.262842		
7.173	0.264170		
7.202	0.328646		
7.2030	0.329875		
7.204	0.330905		
7.205	0.331724		
7.206	0.332322		
7.207	0.332690		
7.208	0.332821		
	(Higher crest)		
7.209	0.332708		
7.210	0.332347		

Table No.5

TIME	ω_r / ω_b	TIME	ω_r / ω_b
o.1	0.300278	4.6	0.294214
o.2	0.235127	4.7	0.328798
o.3	0.290551	4.8	0.290463
o.4	0.348424	4.9	0.261025
o.5	0.295133	5.0	0.299703
o.6	0.243965	5.1	0.328130
o.7	0.294955	5.2	0.284116
o.8	0.342765	5.3	0.263109
o.9	0.289327	5.4	0.305039
1.o	0.249668	5.5	0.326365
1.1	0.299779	5.6	0.278234
1.2	0.338750	5.7	0.266011
1.3	0.283521	5.8	0.310103
1.4	0.254312	5.9	0.323515
1.5	0.304656	6.0	0.273017
1.6	0.335066	6.1	0.269629
1.7	0.278005	6.2	0.314766
1.8	0.258686	6.3	0.319639
1.9	0.309417	6.4	0.268638
2.0	0.331152	6.5	0.273850
.1	0.272970	6.6	0.318891
2.2	0.263128	6.7	0.314852
2.3	0.313935	6.8	0.265230
2.4	0.326749	6.9	0.278556
2.5	0.268572	7.0	0.322335
2.6	0.267782	7.099	0.309312
2.7	0.318086	7.100	0.306736
2.8	0.321759	7.101	0.304067
2.9	0. 64947	7.102	0.301325
3.0	0.272692	7.103	0.298532
3.1	0.321741	7.104	0.295709
3.2	0.316186	7.105	0.292877
3.3	0.262204	7.106	0.290059
3.4	0.277845	7.107	0.287276
3.5	0.324765	7.108	0.284549
3.6	0.310112	7.109	0.281899
3.7	0.260420	7.110	0.279344
3.8	0.283197	7.111	0.276905
3.9	0.327028	7.112	0.274597
4.0	0.303675	7.113	0.272437
4.1	0.259633	7.114	0.270440
4.2	0.288680	7.115	0.268617
4.3	0.328405	7.116	0.266980
4.4	0.297056	7.117	0.265537
4.5	0.259846	7.118	0.264297

TIME	ω_r/ω_b	TIME	ω_r/ω_b
7.119	0.263264	7.245	0.327499
7.120	0.26[illegible]442		Higher Crest
7.121	0.261834	7.246	0.327357
7.122	0.261438	7.247	0.326995
7.123	0.261254	7.248	0.326412
	Lower Crest	7.249	0.325610
7.124	0.261278	7.250	0.324589
7.125	0.261506	7.251	0.323355
7.126	0.261933	7.252	0.321913
7.127	0.262552	7.253	0.320273
7.128	0.263355	7.274	0.268794
7.129	0.264333	7.275	0.267143
7.152	0.311524	7.276	0.265686
7.153	0.313576	7.277	0.264431
7.154	0.315535	7.278	0.263382
7.155	0.317387	7.279	0.262544
7.156	0.319121	7.280	0.261919
7.157	0.320724	7.281	0.261506
7.158	0.322184	7.282	0.261305
7.159	0.323491		Lower Crest
7.160	0.324632		
7.161	0.3[illegible]5597		
7.162	0.326376		
7.163	0.326960		
7.164	0.327342		
7.165	0.327514		
	Higher Crest		
7.166	0.327472		
7.167	0.3[illegible]7211		
7.168	0.326730		
7.169	0.326028		
7.170	0.325106		
7.171	0.323969		
7.200	0.262158		
7.201	0.261648		
7.202	0.261350		
7.203	0.261262		
	Lower Crest		
7.204	0.261380		
7.205	0.261700		
7.206	0.262215		
7.240	0.325078		
7.241	0.325958		
7.242	0.326648		
7.243	0.3[illegible]7139		
7.244	0.327425		

Table No. 6

Time	ω_r/ω_b	Time	ω_r/ω_b
0.1	0.334522	4.1	0.336564
0.2	0.326519	4.2	0.382138
0.3	0.384933	4.3	0.326776
0.4	0.334175	4.4	0.337389
0.5	0.327266	4.5	0.381942
0.6	0.384524	4.6	0.326146
0.7	0.333807	4.7	0.338222
0.8	0.328013	4.8	0.381729
0.9	0.384190	4.9	0.325524
1.0	0.333372	5.0	0.339063
1.1	0.328761	5.1	0.381499
1.2	0.328711	5.2	0.324912
1.3	0.332884	5.3	0.339912
1.4	0.329512	5.4	0.381248
1.5	0.383676	5.5	0.324313
1.6	0.332353	5.6	0.340769
1.7	0.330267	5.7	0.380977
1.8	0.383472	5.8	0.323728
1.9	0.331789	5.9	0.341632
2.0	0.331028	6.0	0.380684
2.1	0.383290	6.1	0.323159
2.2	0.331200	6.2	0.342502
2.3	0.331795	6.3	0.380368
2.4	0.383123	6.4	0.322606
2.5	0.330590	6.5	0.343378
2.6	0.332569	6.6	0.380027
2.7	0.382965	6.7	0.322072
2.8	0.329967	6.8	0.344259
2.9	0.333352	6.9	0.379663
3.0	0.382820	7.0	0.321558
3.1	0.329338	7.099	0.345146
3.2	0.334142	7.100	0.347704
3.3	0.382653	7.101	0.350273
3.4	0.328694	7.102	0.352840
3.5	0.334941	7.103	0.355388
3.6	0.382491	7.104	0.357904
3.7	0.328053	7.105	0.360372
3.8	0.335748	7.106	0.362777
3.9	0.382321	7.107	0.365104
4.0	0.327413	7.108	0.367337

Time	ω_r/ω_b
7.109	0.369461
7.110	0.371461
7.111	0.373321
7.112	0.375027
7.113	0.376565
7.114	0.377921
7.115	0.379083
7.116	0.380039
7.117	0.380779
7.118	0.381293
7.119	0.381576
7.120	0.381620
	(Higher Crest)
7.121	0.381423
7.122	0.380982
7.123	0.380297
7.124	0.379373
7.125	0.378212
7.126	0.376823
7.127	0.375213
7.128	0.373396
7.129	0.371384
7.130	0.369192
7.148	0.322918
7.149	0.321308
7.150	0.319908
7.151	0.318726
7.152	0.317767
7.153	0.317036
7.154	0.316533
7.155	0.316260
7.156	0.316215
	(Lower Crest)
7.157	0.316394
7.158	0.016793
7.159	0.317405
7.175	0.347927
7.176	0.350496
7.177	0.353061
7.178	0.355607
7.179	0.358118

Time	ω_r/ω_b
7.180	0.360581
7.181	0.362979
7.182	0.365298
7.183	0.367522
7.184	0.369635
7.185	0.371622
7.186	0.373469
7.187	0.375161
7.188	0.376683
7.189	0.378022
7.190	0.379166
7.191	0.380104
7.192	0.380824
7.193	0.381319
7.194	0.381580
7.195	0.381604
	(Higher Crest)
7.196	0.381385
7.197	0.380923
7.198	0.380218
7.199	0.379278
7.200	0.378092
7.201	0.376684
7.202	0.375056
7.203	0.373222
7.204	0.371194
7.225	0.319804
7.226	0.318642
7.227	0.317703
7.228	0.316991
7.229	0.316509
7.230	0.316256
7.231	0.316230
	(Lower Crest)
7.232	0.316428
7.233	0.316846
7.234	0.317476
7.235	0.318311
7.236	0.319344
7.237	0.320564
7.238	0.321961

Table No. 7

Time	ω_r/ω_b	Time	ω_r/ω_b
0.1	0.334704	4.0	0.321253
0.2	0.323123	4.1	0.346640
0.3	0.381476	4.2	0.373612
0.4	0.339284	4.3	0.322746
0.5	0.321155	4.4	0.344177
0.6	0.379869	4.5	0.374448
0.7	0.343592	4.6	0.324412
0.8	0.319589	4.7	0.341788
0.9	0.376206	4.8	0.375043
1.0	0.347583	4.9	0.326234
1.1	0.318378	5.0	0.339484
1.2	0.373502	5.1	0.375401
1.3	0.351280	5.2	0.328199
1.4	0.317492	5.3	0.337273
1.5	0.370771	5.4	0.375530
1.6	0.354695	5.5	0.330291
1.7	0.316906	5.6	0.335162
1.8	0.368023	5.7	0.375438
1.9	0.357838	5.8	0.332494
2.0	0.316602	5.9	0.333162
2.1	0.365270	6.0	0.375131
2.2	0.360714	6.1	0.334792
2.3	0.316564	6.2	0.331280
2.4	0.362522	6.3	0.374620
2.5	0.363328	6.4	0.337165
2.6	0.316788	6.5	0.329524
2.7	0.365681	6.6	0.373914
2.8	0.317236	6.7	0.339597
2.9	0.355707	6.8	0.327903
3.0	0.367776	6.9	0.373024
3.1	0.317923	7.099	0.342068
3.2	0.354398	7.100	0.339737
3.3	0.369615	7.101	0.337469
3.4	0.318829	7.102	0.335281
3.5	0.351760	7.103	0.333189
3.6	0.371199	7.104	0.331211
3.7	0.319943	7.105	0.329358
3.8	0.349171	7.106	0.327646
3.9	0.372530	7.107	0.326084

Time	ω_r/ω_b
7.108	0.324684
7.109	0.323454
7.110	0.322401
7.111	0.321531
7.112	0.320848
7.113	0.320353
7.114	0.320049
7.115	0.319935
	(Lower Crest)
7.116	0.320008
7.117	0.320267
7.118	0.320706
7.119	0.321321
7.120	0.322105
7.121	0.323051
7.122	0.324152
7.145	0.367534
7.146	0.368998
7.147	0.370323
7.148	0.371498
7.149	0.372513
7.150	0.373358
7.151	0.374026
7.152	0.374508
7.153	0.374999
7.154	0.374894
	(Higher Crest)
7.155	0.374790
7.156	0.374486
7.157	0.373981
7.182	0.326496
7.183	0.325054
7.184	0.323780
7.185	0.322681
7.186	0.321764
7.187	0.321032
7.188	0.320488
7.189	0.320135
7.190	0.319971
	(Lower Crest)
7.191	0.319995
7.192	0.320206
7.193	0.320598
7.194	0.321167
7.195	0.321906
7.196	0.322810
7.197	0.323870
7.198	0.325078
7.199	0.326424
7.224	0.372242
7.225	0.373132
7.226	0.373846
7.227	0.374376
7.228	0.374717
7.229	0.374864
	(Higher Crest)
7.230	0.374812

Table No. 8

Time	ω_r/ω_b	Time	ω_r/ω_b
0.1	0.334989	4.0	0.362199
0.2	0.319816	4.1	0.322797
0.3	0.377161	4.2	0.349424
0.4	0.344801	4.3	0.365357
0.5	0.316303	4.4	0.324898
0.6	0.370848	4.5	0.344568
0.7	0.353529	4.6	0.367429
0.8	0.314704	4.7	0.327925
0.9	0.364047	4.8	0.339947
1.0	0.360808	4.9	0.368384
1.1	0.314882	5.0	0.331719
1.2	0.357111	5.1	0.335719
1.3	0.366463	5.2	0.368250
1.4	0.316677	5.3	0.336086
1.5	0.350317	5.4	0.332027
1.6	0.370414	5.5	0.367102
1.7	0.319886	5.6	0.340806
1.8	0.343892	5.7	0.328996
1.9	0.372674	5.8	0.365056
2.0	0.324263	5.9	0.345650
2.1	0.338023	6.0	0.326730
2.2	0.373333	6.1	0.362252
2.3	0.329521	6.2	0.350377
2.4	0.332870	6.3	0.325309
2.5	0.372544	6.4	0.358852
2.6	0.335349	6.5	0.354763
2.7	0.328569	6.6	0.324785
2.8	0.370502	6.7	0.355023
2.9	0.341422	6.8	0.358612
3.0	0.325233	6.9	0.325177
3.1	0.367429	7.099	0.350935
3.2	0.322951	7.100	0.352536
3.3	0.363551	7.101	0.354098
3.4	0.353049	7.102	0.356111
3.5	0.321777	7.103	0.357064
3.6	0.359119	7.104	0.358449
3.7	0.358046	7.105	0.359755
3.8	0.321733	7.106	0.360973
3.9	0.354339	7.107	0.362094

Time	ω_r/ω_b
7.108	0.363111
7.109	0.364014
7.110	0.364797
7.111	0.365452
7.112	0.365974
7.113	0.366358
7.114	0.366600
7.115	0.366696
	(Higher Crest)
7.116	0.366645
7.117	0.366445
7.118	0.366096
7.119	0.365600
7.120	0.364960
7.121	0.364179
7.122	0.363262
7.145	0.328525
7.146	0.327613
7.147	0.326843
7.148	0.326208
7.149	0.325712
7.150	0.325357
7.151	0.325144
7.152	0.325073
	(Lower Crest)
7.153	0.325143
7.154	0.325352
7.155	0.325697
7.156	0.326174
7.157	0.326780
7.158	0.327509
7.159	0.328355
7.182	0.361354
7.183	0.362435
7.184	0.363408

Time	ω_r/ω_b
7.185	0.364266
7.186	0.365001
7.187	0.365607
7.188	0.366078
7.189	0.366409
7.190	0.366598
7.191	0.366640
	(Higher Crest)
7.192	0.366534
7.193	0.366280
7.194	0.365879
7.195	0.365331
7.196	0.364641
7.222	0.327365
7.223	0.326645
7.224	0.326062
7.225	0.325619
7.226	0.325316
7.227	0.325155
7.228	0.325136
	(Lower Crest)
7.229	0.325256
7.230	0.325514
7.231	0.325907
7.257	0.360578
7.258	0.361721
7.259	0.362761
7.260	0.363690
7.261	0.364501
7.262	0.365188
7.263	0.365743
7.264	0.366163
7.265	0.366441
7.266	0.366576
	(Higher Crest)

Table No. 9

Time	ω_r/ω_b	Time	ω_r/ω_b
0.1	0.335369	3.8	0.357867
0.2	0.316667	3.9	0.350229
0.3	0.371983	4.0	0.326146
0.4	0.350319	4.1	0.355235
0.5	0.313199	4.2	0.355740
0.6	0.360983	4.3	0.327125
0.7	0.361810	4.4	0.346359
0.8	0.314492	4.5	0.359382
0.9	0.349736	4.6	0.330133
1.0	0.368806	4.7	0.340554
1.1	0.319804	4.8	0.360867
1.2	0.339454	4.9	0.334721
1.3	0.371214	5.0	0.335537
1.4	0.327990	5.1	0.360220
1.5	0.331007	5.2	0.340243
1.6	0.369591	5.3	0.331787
1.7	0.337619	5.4	0.357729
1.8	0.325024	5.5	0.345953
1.9	0.364886	5.6	0.329649
2.0	0.347201	5.7	0.353858
2.1	0.321914	5.8	0.351118
2.2	0.358194	5.9	0.329299
2.3	0.355413	6.0	0.349162
2.4	0.321818	6.1	0.355119
2.5	0.350579	6.2	0.330717
2.6	0.361293	6.3	0.344216
2.7	0.324563	6.4	0.357544
2.8	0.342982	6.5	0.333682
2.9	0.364340	6.6	0.339560
3.0	0.329641	6.7	0.358218
3.1	0.336192	6.8	0.337785
3.2	0.364514	6.9	0.335669
3.3	0.336258	7.099	0.342479
3.4	0.330842	7.100	0.341359
3.5	0.362158	7.101	0.340262
3.6	0.343450	7.102	0.339193
3.7	0.327401	7.103	0.338163

Time	ω_r/ω_b
7.104	0.337178
7.105	0.336245
7.106	0.335370
7.107	0.334560
7.108	0.333820
7.109	0.333154
7.110	0.332567
7.111	0.332062
7.112	0.331642
7.113	0.331310
7.114	0.331068
7.115	0.330916
7.116	0.330854
	(Lower Crest)
7.117	0.330883
7.118	0.331002
7.119	0.331209
7.120	0.331502
7.121	0.331880
7.122	0.332338
7.123	0.332873
7.124	0.333483
7.125	0.334162
7.126	0.334905
7.127	0.335709
7.149	0.356135
7.150	0.356608
7.151	0.356998
7.152	0.357302
7.153	0.357518
7.154	0.357642
7.155	0.357674
	(Higher Crest)
7.156	0.357613
7.157	0.357459
7.158	0.357212
7.159	0.356873
7.160	0.356445

Time	ω_r/ω_b
7.183	0.334663
7.184	0.333924
7.185	0.333258
7.186	0.332671
7.187	0.332165
7.188	0.331744
7.189	0.331410
7.190	0.331164
7.191	0.331008
7.192	0.330943
	(Lower Crest)
7.193	0.330967
7.227	0.356898
7.228	0.357204
7.229	0.347422
7.230	0.357550
7.231	0.357586
	(Higher Crest)
7.232	0.357530

Table No.10

TIME	ω_r / ω_b	TIME	ω_r / ω_b
0.1	0.282566	4.3	0.256515
0.2	0.252892	4.4	0.357377
0.3	0.354139	4.5	0.298375
0.4	0.303760	4.6	0.244551
0.5	0.242788	4.7	0.345604
0.6	0.342012	4.8	0.320509
0.7	0.325245	4.9	0.236649
0.8	0.235852	5.0	0.330943
0.9	0.327153	5.1	0.340291
1.0	0.344020	5.2	0.234127
1.1	0.234699	5.3	0.314667
1.2	0.310805	5.4	0.355436
1.3	0.357855	5.5	0.237927
1.4	0.239875	5.6	0.297747
1.5	0.293939	5.7	0.364404
1.6	0.365355	5.8	0.248269
1.7	0.251527	5.9	0.230998
1.8	0.277377	6.0	0.366632
1.9	0.366147	6.1	0.264443
2.0	0.268743	6.2	0.265236
2.1	0.261983	6.3	0.362532
2.2	0.360812	6.4	0.284827
2.3	0.289723	6.5	0.251462
2.4	0.248817	6.6	0.353229
2.5	0.350573	6.7	0.307105
2.6	0.312042	6.8	0.240915
2.7	0.239171	6.9	0.340167
2.8	0.336891	7.0	0.328628
2.9	0.333028	7.099	0.234963
3.0	0.[illegible]34412	7.100	0.236063
3.1	0.321120	7.101	0.237580
3.2	0.350205	7.102	0.239490
3.3	0.235675	7.103	0.241765
3.4	0.304351	7.104	0.244376
3.5	0.361700	7.105	0.247295
3.6	0.245497	7.106	0.250494
3.7	0.287446	7.107	0.253942
3.8	0.366554	7.108	0.257613
3.9	0.257543	7.109	0.261480
4.0	0.271196	7.110	0.265517
4.1	0.364817	7.111	0.269700
4.2	0.276530	7.112	0.274008

TIME	ω_r/ω_b	TIME	ω_r/ω_b
7.113	0.278419	7.157	0.291531
7.114	0.282913	7.158	0.285484
7.115	0.287474	7.159	0.279576
7.116	0.292083	7.160	00.273859
7.117	0.296725	7.161	0.268383
7.118	0.301382	7.162	0.263195
7.119	0.306040	7.163	0.258339
7.120	0.310683	7.164	0.253853
7.121	0.315293	7.165	0.249772
7.122	0.319854	7.166	0.246126
7.123	0.324347	7.167	0.242938
7.124	0.328753	7.168	0.240228
7.125	0.333049	7.169	0.238009
7.126	0.337213	7.170	0.236287
7.127	0.341220	7.171	0.235065
7.128	0.345044	7.172	0.234341
7.129	0.348656	7.173	0.<u>234105</u>
7.130	0.352027		Lower Crest
7.131	0.355127	7.174	0.234345
7.132	0.357926	7.175	0.235044
7.133	0.360391	7.176	0.236182
7.134	0.362492	7.177	0.237735
7.135	0.364201	7.178	0.239678
7.136	0.365489	7.179	0.241984
7.137	0.366330	7.180	0.244624
7.138	0.<u>366704</u>	7.181	0.247569
	Higher Crest	7.182	0.250791
7.139	0.366591	7.183	0.254260
7.140	0.365978	7.184	0.257950
7.141	0.364855	7.185	0.261832
7.142	0.363218	7.186	0.265883
7.143	0.361070	7.187	0.270079
7.144	0.358419	7.188	0.274396
7.145	0.355277	7.189	0.278816
7.146	0.351666	7.190	0.283317
7.147	0.347610	7.191	0.287883
7.148	0.343140	7.192	0.292495
7.149	0.338293	7.193	0.297139
7.150	0.333109	7.194	0.301797
7.151	0.327634	7.195	0.306454
7.152	0.321915	7.196	0.311095
7.153	0.316004	7.197	0.315701
7.154	0.309955	7.198	0.320257
7.155	0.303822	7.199	0.324743
7.156	0.297662	7.200	0.333426

TIME	ω_r/ω_b	TIME	ω_r/ω_b
7.201	0.333426	7.243	0.242677
7.202	0.333577	7.244	0.240010
7.203	0.341568	7.245	0.237835
7.204	0.345374	7.246	0.236158
7.205	0.348966	7.247	0.234981
7.206	0.352315	7.248	0.234300
7.207	0.355389	7.249	0.234107
7.208	0.358159		Lower Crest
7.209	0.360593	7.250	0.234389
7.210	0.362661	7.251	0.235127
7.211	0.364333	7.252	0.236303
7.212	0.365582	7.253	0.237892
7.213	0.366383	7.254	0.239869
7.214	0.366714	7.255	0.242206
	Higher Crest	7.256	0.244874
7.215	0.366557	7.257	0.247845
7.216	0.365899	7.258	0.251090
7.217	0.364730	7.259	0.254580
7.218	0.363048	7.260	0.258288
7.219	0.360854	7.261	0.262186
7.220	0.358159	7.262	0.266251
7.221	0.354975	7.263	0.270458
7.222	0.351322	7.264	0.274786
7.223	0.347228	7.265	0.279213
7.224	0.342723	7.266	0.283721
7.225	0.337845	7.267	0.288292
7.226	0.332633	7.268	0.292908
7.227	0.327134	7.269	0.297553
7.228	0.321396	7.270	0.302212
7.229	0.315471	7.271	0.306868
7.230	0.309412	7.272	0.311506
7.231	0.303274	7.273	0.316109
7.232	0.297115	7.274	0.320660
7.233	0.290989	7.275	0.325139
7.234	0.284952	7.276	0.329526
7.235	0.279058	7.277	0.333801
7.236	0.273361	7.278	0.337939
7.237	0.267909	7.279	0.341935
7.238	0.262749	7.280	0.345703
7.239	0.257924	7.281	0.349275
7.240	0.253473	7.282	0.352600
7.241	0.249429	7.283	0.355649
7.242	0.245823	7.284	0.358390

TIME	ω_r/ω_b	TIME	ω_r/ω_b
7.285	0.360792		
7.286	0.362826		
7.287	0.364462		
7.288	0.365672		
7.289	0.366432		
7.290	0.366720		
	Higher Crest		
7.291	0.366519		
7.292	0.365815		

Table No. 11

Time	ω_r/ b	Time	ω_r/ b
0.1	0.282913	4.1	0.317027
0.2	0.250525	4.2	0.350313
0.3	0.349065	4.3	0.237640
0.4	0.310344	4.4	0.296138
0.5	0.238479	4.5	0.361715
0.6	0.331793	4.6	0.250777
0.7	0.335855	4.7	0.275562
0.8	0.234333	4.8	0.362607
0.9	0.311637	4.9	0.272622
1.0	0.354406	5.0	0.256998
1.1	0.240026	5.1	0.354138
1.2	0.290624	5.2	0.299489
1.3	0.363036	5.3	0.242618
1.4	0.255756	0.4	0.338902
1.5	2.270374	5.5	0.326336
1.6	0.361237	5.6	0.235049
1.7	0.279403	5.7	0.319730
1.8	0.252669	5.8	0.348049
1.9	0.350674	5.9	0.236662
2.0	0.306809	6.0	0.298919
2.1	0.239826	6.1	0.360808
2.2	0.334119	6.2	0.248482
2.3	0.332777	6.3	0.278211
2.4	0.234492	6.4	0.363070
2.5	0.314308	6.5	0.269342
2.6	0.352415	6.6	0.259269
2.7	0.238791	6.7	0.355711
2.8	0.293364	6.8	0.295817
2.9	0.362440	6.9	0.244190
3.0	0.253222	7.0	3412
3.1	0.272948	7.099	0.322972
3.2	0.361981	7.100	0.317337
3.3	0.275988	7.101	0.311539
3.4	0.254798	7.102	0.305631
3.5	0.352448	7.103	0.299667
3.6	0.303160	7.104	0.293700
3.7	0.241165	7.105	0.287785
3.8	0.336535	7.106	0.281975
3.9	0.329607	7.107	0.276322
4.0	0.234693	7.108	0.270874

Time	ω_r / b
7.109	0.265680
7.110	0.260782
7.111	0.256221
7.112	0.252031
7.113	0.248246
7.114	0.244889
7.115	0.241983
7.116	0.239543
7.117	0.237579
7.118	0.236096
7.119	0.235094
7.120	0.234568
7.121	0.234508
	(Lower Crest)
7.122	0.234901
7.123	0.235729
7.124	0.236971
7.125	0.238605
7.126	0.240605
7.127	0.242945
7.128	0.245598
7.129	0.248536
7.157	0.358067
7.158	0.359980
7.159	0.361502
7.160	0.362609
7.161	0.363276
7.162	0.363483
	(Higher Crest)
7.163	0.363215
7.164	0.362460
7.165	0.361209
7.166	0.359462
7.190	0.245400
7.191	0.242420
7.192	0.239904
7.193	0.237863
7.194	0.236302
7.195	0.235223
7.196	0.234621
7.197	0.234487
	(Lower Crest)
7.198	0.234808

Time	ω_r / b
7.199	0.235567
7.200	0.236744
7.201	0.238316
7.202	0.240259
7.203	0.242546
7.204	0.245150
7.205	0.248044
7.206	0.251200
7.234	0.359697
7.235	0.361285
7.236	0.362460
7.237	0.363199
7.238	0.363482
	(Higher Crest)
7.239	0.363291
7.240	0.362615
7.241	0.361444
7.242	0.359777
7.243	0.357617
7.244	0.354971
7.245	0.351855
7.268	0.240278
7.269	0.238159
7.270	0.236521
7.271	0.235365
7.272	0.234686
7.273	0.234477
	(Lower Crest)
7.274	0.234726
7.275	0.235416

Table No. 12

Time	W_r/b	Time	W_r/b
0.1	0.283398	4.6	0.235864
0.2	0.247247	4.7	0.304280
0.3	0.343235	4.8	0.354481
0.4	0.317055	4.9	0.246115
0.5	0.235514	5.0	0.279685
0.6	0.230363	5.1	0.359141
0.7	0.344548	5.2	0.269788
0.8	0.236885	5.3	0.257443
0.9	0.295187	5.4	0.349901
1.0	0.253000	5.5	0.301250
1.1	0.270777	5.6	0.241256
1.2	0.357611	5.7	0.331054
1.3	0.280941	5.8	0.331759
1.4	0.250174	5.9	0.235621
1.5	0.343894	6.0	0.307427
1.6	0.313435	6.1	0.352790
1.7	0.237419	6.2	0.244001
1.8	0.322530	6.3	0.282752
1.9	0.341354	6.4	0.359381
2.0	0.236889	6.5	0.266182
2.1	0.298027	6.6	0.260029
2.2	0.257223	6.7	0.351718
2.3	0.250887	6.8	0.297100
2.4	0.273680	6.9	0.242827
2.5	0.258063	7.0	0.247827
2.6	0.277312	7.099	0.333809
2.7	0.252554	7.100	0.337461
2.8	0.345865	7.101	0.340927
2.9	0.309465	7.102	0.344181
3.0	0.238611	7.103	0.347195
3.1	0.325345	7.104	0.349942
3.2	0.338306	7.105	0.352393
3.3	0.236322	7.106	0.354520
3.4	0.301131	7.107	0.356296
3.5	0.355938	7.108	0.357695
3.6	0.248438	7.109	0.358693
3.7	0.276657	7.110	0.359269
3.8	0.358682	7.111	0.359404
3.9	0.273517		(Higher Crest)
4.0	0.254953	7.112	0.359085
4.1	0.347935	7.113	0.358299
4.2	0.305382	7.114	0.357043
4.3	0.239853	7.115	0.355314
4.4	0.328220	7.116	0.353118
4.5	0.335116	7.117	0.350463

Time	ω_r/b
7.139	0.246052
7.140	0.243210
7.141	0.240808
7.142	0.238856
7.143	0.237360
7.144	0.236321
7.145	0.235734
7.146	0.235591
	(Lower Crest)
7.147	0.235880
7.148	0.236585
7.149	0.237687
7.150	0.239165
7.151	0.240997
7.152	0.243158
7.153	0.245623
7.182	0.353652
7.183	0.355581
7.184	0.357144
7.185	0.358317
7.186	0.359075
7.187	0.359401
	(High Crest)
7.188	0.359278
7.189	0.358693
7.190	0.357638
7.191	0.561
7.192	0.354114
7.193	0.351654
7.194	0.348744
7.195	0.345401
7.196	0.341649
7.197	0.337516
7.198	0.333036
7.199	0.2447
7.217	0.241781
7.218	0.239635
7.219	0.237944
7.220	0.236710

Time	ω_r/b
7.221	0.235930
7.222	0.235599
	(Lower Crest)
7.223	0.235704
7.224	0.236233
7.225	0.237268
7.226	0.238488
7.227	0.240172
7.228	0.242195
7.229	0.244533
7.230	0.247160
7.231	0.250050
7.232	0.253178
7.233	0.256519
7.234	0.250049
7.235	0.263745
7.236	0.267585
7.237	0.271550
7.259	0.354799
7.260	0.356522
7.261	0.357864
7.262	0.358803
7.263	0.389316
7.264	0.359387
	(Higher Crest)
7.265	0.359000
7.266	0.358147
7.267	0.356823

Table No. 13

Time	ω_r/b	Time	ω_r/b
0.1	0.284101	4.1	0.243319
0.2	0.244044	4.2	0.284029
0.3	0.336623	4.3	0.354428
0.4	0.323617	4.4	0.267354
0.5	0.234305	4.5	0.258442
0.6	0.307993	4.6	0.345033
0.7	0.350124	4.7	0.302556
0.8	0.244298	4.8	0.241070
0.9	0.278696	4.9	0.332824
1.0	0.355671	5.0	0.335285
1.1	0.272788	5.1	0.238825
1.2	0.253519	5.2	0.295306
1.3	0.342334	5.3	0.352959
1.4	0.309800	5.4	0.255710
1.5	0.238423	5.5	0.268150
1.6	0.317839	5.6	0.357798
1.7	0.340853	5.7	0.287819
1.8	0.240092	5.8	0.246655
1.9	0.289586	5.9	0.332808
2.0	0.354571	6.0	0.323294
2.1	0.261062	6.1	0.237527
2.2	0.262914	6.2	0.306648
2.3	0.348416	6.3	0.348161
2.4	0.295340	6.4	0.246636
2.5	0.243310	6.5	0.278755
2.6	0.327910	6.6	0.353951
2.7	0.329862	6.7	0.273655
2.8	0.237638	6.8	0.254253
2.9	0.300808	6.9	0.341489
3.0	0.351187	7.0	0.309508
3.1	0.250891	7.099	0.239271
3.2	0.273131	7.100	0.240590
3.3	0.352743	7.101	0.242243
3.4	0.280865	7.102	0.244209
3.5	0.250031	7.103	0.246465
3.6	0.337203	7.104	0.248987
3.7	0.316880	7.105	0.251753
3.8	0.237955	7.106	0.254738
3.9	0.312016	7.107	0.257921
4.0	0.344740	7.108	0.261279
		7.109	0.264791

Time	ω_r/b
7.110	0.268439
7.111	0.272202
7.112	0.276063
7.113	0.280006
7.114	0.284013
7.115	0.288070
7.116	0.292161
7.117	0.296272
7.118	0.300388
7.119	0.304493
7.120	0.308573
7.121	0.312612
7.122	0.316591
7.123	0.320493
7.124	0.324298
7.125	0.327986
7.126	0.331535
7.127	0.334922
7.128	0.338123
7.129	0.341112
7.130	0.343864
7.131	0.346352
7.132	0.348552
7.133	0.350437
7.134	0.351983
7.135	0.353166
7.136	0.353966
7.137	0.354365
	(Higher Crest)
7.138	0.354346
7.139	0.353898
7.140	0.353013
7.1419	0.351688
7.142	0.349923
7.143	0.347724
7.166	0.246931
7.167	0.244333
7.168	0.242147
7.169	0.240380
7.170	0.239038
7.171	0.238119
7.172	0.237621

Time		ω_r/b
7.173		0.237535
		(Lower Crest)
7.174		0.237849
7.200		0.321605
7.201		0.325378
7.202		0.329029
7.211		0.352363
7.212		0.353438
7.213		0.354123
7.214	Higher	0.354403
7.215	Crest	0.354261
7.216		0.353687
7.246		0.239948
7.247		0.238728
7.248		0.237932
7.249		0.237554
		(Lower Crest)
7.250		0.237585
7.251		0.238012
7.252		0.238821

Table No. 14

Time	ω_r / b	Time	ω_r / b
0.1	0.284964	4.1	0.313712
0.2	0.241034	4.2	0.240548
0.3	0.329291	4.3	0.306239
0.4	0.329488	4.4	0.342198
0.5	0.235327	4.5	0. 249593
0.6	0.295163	4.6	0.275893
0.7	0.351431	4.7	0.347231
0.8	0.256565	4.8	0.280433
0.9	0.263504	4.9	0.250814
1.0	0.345239	5.0	0.330335
1.1	0.296043	5.1	0.318730
1.2	0.242203	5.2	0.240395
1.3	0.320362	5.3	0.301964
1.4	0.333207	5.4	0.344401
1.5	0.240943	5.5	0.252813
1.6	0.289009	5.6	0.271891
1.7	0.349444	5.7	0.345962
1.8	0.264238	5.8	0.284803
1.9	0.259960	5.9	0.248261
2.0	0.347451	6.0	0.326795
2.1	0.302755	6.1	0.323473
2.2	0.241760	6.2	0.240672
2.3	0.315040	6.3	0.297684
2.4	0.336525	6.4	0.346124
2.5	0.244037	6.5	0.256426
2.6	0.284313	6.6	0.268016
2.7	0.348744	6.7	0.344303
2.8	0.269894	6.8	0.291251
2.9	0.256697	6.9	0.246016
3.0	0.336977	7.0	0.323083
3.1	0.308427	7.099	0.327903
3.2	0.241059	7.100	0.323960
3.3	0.310543	7.101	0.319766
3.4	0.339552	7.102	0.315355
3.5	0.246726	7.103	0.310766
3.6	0.280018	7.104	0.306037
3.7	0.348123	7.105	0.301210
3.8	0.275158	7.106	0.296327
3.9	0.253639	7.107	0.291431
4.0	0.333705	7.108	0.286564

TABLE NO. 15

TIME	ω_r/ω_b	TIME	ω_r/ω_b
0.1	0.329906	4.3	0.416143
0.2	0.429433	4.4	0.431021
0.3	0.412256	4.5	0.333934
0.4	0.346947	4.6	0.496762
0.5	0.491899	4.7	0.310668
0.6	0.308326	4.8	0.451163
0.7	0.465654	4.9	0.386807
0.8	0.366522	5.0	0.363499
0.9	0.380009	5.1	0.483240
1.0	0.469391	5.2	0.308498
1.1	0.313727	5.3	0.480089
1.2	0.489764	5.4	0.346525
1.3	0.330769	5.5	0.398099
1.4	0.416021	5.6	0.451545
1.5	0.431100	5.7	0.322335
1.6	0.333919	5.8	0.495794
1.7	0.496730	5.9	0.318453
1.8	0.310703	6.0	0.434036
1.9	0.451130	6.1	0.409026
2.0	0.386834	6.2	0.347844
2.1	0.363489	6.3	0.492557
2.2	0.483240	6.4	0.307385
2.3	0.308502	6.5	0.466786
2.4	0.460031	6.6	0.365594
2.5	0.346583	6.7	0.380403
2.6	0.398095	6.8	0.469282
2.7	0.451518	6.9	0.313655
2.8	0.322333	7.0	0.489997
2.9	0.495793	7.099	0.330562
3.0	0.318454	7.100	0.325098
3.1	0.434033	7.101	0.320323
3.2	0.409030	7.102	0.316267
3.3	0.347842	7.103	0.312954
3.4	0.492558	7.104	0.310396
3.5	0.307386	7.105	0.308598
3.6	0.466786	7.106	0.307558
3.7	0.365597	7.107	0.307265
3.8	0.380400		(Lower Crest)
3.9	0.469284	7.108	0.307700
4.0	0.313654	7.109	0.308840
4.1	0.489996	7.110	0.310653
4.2	0.330665	7.111	0.318105

TIME	ω_r/ω_b	TIME	ω_r/ω_b
7.132	0.443254	7.216	0.496808
7.133	0.450172	7.217	0.495780
7.134	0.456847	7.218	0.493873
7.135	0.463221	7.219	0.491093
7.136	0.469234	7.220	0.487460
7.137	0.474823	7.221	0.483003
7.138	0.479923	7.244	0.314395
7.139	0.484470	7.245	0.311486
7.140	0.488401	7.246	0.309336
7.141	0.491657	7.247	0.307946
7.142	0.494183	7.248	0.307309
7.143	0.495929		(Lower Crest)
7.144	0.496857	7.249	0.307409
7.145	0.496934	7.250	0.308225
	(Higher Crest)	7.251	0.309730
7.146	0.496142	7.252	0.311891
7.147	0.494470	7.253	0.314671
7.148	0.491923	7.254	0.318031
7.149	0.488516	7.280	0.482436
7.150	0.484277	7.281	0.486660
7.171	0.323747	7.282	0.490236
7.172	0.319162	7.283	0.493107
7.173	0.315304	7.284	0.495220
7.174	0.312192	7.285	0.496532
7.175	0.309838	7.286	0.497006
7.176	0.308245		(Higher Crest)
7.177	0.307407	7.287	0.496618
7.178	0.307311	7.288	0.495355
	(Lower Crest)		
7.179	0.307938		
7.180	0.309261		
7.181	0.311249		
7.182	0.313867		
7.183	0.317075		
7.206	0.464873		
7.207	0.470778		
7.208	0.476242		
7.209	0.481200		
7.210	0.485588		
7.211	0.489344		
7.212	0.492409		
7.213	0.494730		
7.214	0.496260		
7.215	0.496963		
	(Higher Crest)		

TABLE NO. 16

TIME	ω_r/ω_b	TIME	ω_r/ω_b
0.1	0.327242	4.3	0.319622
0.2	0.421628	4.4	0.487857
0.3	0.422802	4.5	0.329688
0.4	0.337355	4.6	0.415414
0.5	0.491739	4.7	0.430749
0.6	0.314051	4.8	0.332497
0.7	0.443684	4.9	0.492143
0.8	0.395350	5.0	0.316604
0.9	0.355088	5.1	0.437201
1.0	0.485754	5.2	0.403864
1.1	0.309920	5.3	0.349216
1.2	0.463184	5.4	0.488576
1.3	0.368870	5.5	0.310401
1.4	0.375212	5.6	0.457406
1.5	0.471907	5.7	0.376962
1.6	0.312646	5.8	0.368664
1.7	0.478872	5.9	0.477073
1.8	0.345525	6.0	0.311080
1.9	0.396786	6.1	0.474439
2.0	0.451523	6.2	0.352438
2.1	0.321354	6.3	0.389863
2.2	0.489025	6.4	0.458567
2.3	0.327183	6.5	0.318016
2.4	0.418909	6.6	0.486517
2.5	0.426601	6.7	0.332350
2.6	0.334916	6.8	0.411910
2.7	0.492116	6.9	0.434835
2.8	0.315157	7.0	0.330173
2.9	0.440540	7.099	0.491972
3.0	0.399545	7.100	0.492112
3.1	0.352138		(Higher Crest)
3.2	0.487272	7.101	0.491427
3.3	0.310059	7.102	0.489906
3.4	0.460356	7.103	0.487553
3.5	0.372866	7.104	0.484380
3.6	0.371923	7.125	0.331808
3.7	0.474576	7.126	0.326631
3.8	0.311784	7.127	0.322113
3.9	0.476726	7.128	0.318283
4.0	0.348919	7.129	0.315162
4.1	0.393317	7.130	0.312760
4.2	0.455106	7.131	0.311082

TIME	ω_r/ω_b	TIME	ω_r/ω_b
7.132	0.310124	7.240	0.491070
7.133	0.309877 (Lower Crest)	7.241	0.491990
7.134	0.310321	7.242	0.492103 (Higher Crest)
7.135	0.311435	7.243	0.491390
7.136	0.313188	7.244	0.489843
7.137	0.315550	7.245	0.487462
7.161	0.459570		
7.162	0.465337		
7.163	0.470698		
7.164	0.475594		
7.165	0.479963		
7.166	0.483747		
7.167	0.486889		
7.168	0.489338		
7.169	0.491048		
7.170	0.491981		
7.171	0.492108 (Higher Crest)		
7.172	0.491408		
7.173	0.489874		
7.174	0.487508		
7.175	0.484322		
7.176	0.480341		
7.177	0.475601		
7.178	0.470146		
7.200	0.315116		
7.201	0.312726		
7.202	0.311060		
7.203	0.310115		
7.204	0.309878		
7.205	0.310334		
7.206	0.311458		
7.207	0.313222		
7.208	0.315593		
7.209	0.318535		
7.232	0.459667		
7.233	0.465428		
7.234	0.470783		
7.235	0.475670		
7.236	0.480030		
7.237	0.483804		
7.238	0.486935		
7.239	0.489372		

CHAPTER 6

OSCILLATION AMPLITUDE FROM LINEARIZED MODEL

6.1 INTRODUCTION

In this chapter, a method is suggested to estimate the amplitude of speed oscillation of the machine, from the linearized model. The oscillation amplitude found from nonlinear model will be same as that found from linearized model if the perturbations are small. A transfer function relating $[\Delta\omega(s)/\Delta T_L(s)]$ is found for the linearized model which is further used to find out residue of the pair of poles which lie on the $j\omega$ axis or on the right hand side of the $j\omega$ axis. The residue will give estimate of oscillation amplitude with fair accuracy. Mathematically, it can be shown as below

$$[\Delta\omega(s)/\Delta T_L(s)] = F(s)/R(s) \tag{6.1}$$

R(s) can be expressed in terms of eigenvalues of the system matrix A, as follows

$$R(s) = K(s - \sigma_1 + j\omega_1)(s-\sigma_1-j\omega_1)(s+\sigma_2-j\omega_2)$$
$$(s+\sigma_2+ j\omega_2)(s+\sigma_3) \quad \text{(from Chapter 5)} \tag{6.2}$$

Thus

$$[\Delta\omega(s)/ \Delta T_L(s)] = [F(s)/R(s)] = [(a_1+jb_1)/(s-\sigma_1+j\omega_1)] +$$
$$[(a_1-jb_1)/(s-\sigma_1-j\omega_1)] + [A_3/(s+\sigma_3)] +$$
$$[(a_2jb_2)/(s+\sigma_2+j\omega_2)] + [(a_2-jb_2)/(s+s_2+jl_2)]$$
$$(6.3)$$

$$\Delta\omega(t)/\Delta T_L = (a_1+jb_1)\ e^{(\sigma_1-j\omega_1)} + (a_1-jb_1)\ e^{(\sigma_1+j\omega_1)t}$$

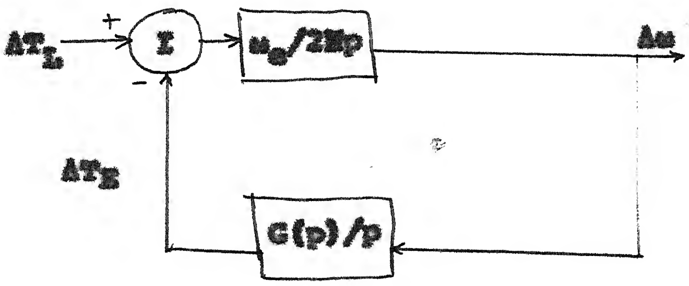
$$+ (a_2+jb_2)\ e^{-(\sigma_2-j\omega_2)t} + (a_2-jb_2)e^{-(\sigma_2+j\omega_2)t}$$

$$+ a_3\ e^{-\sigma_3 t} \qquad (6.4)$$

In steady state only first and second terms will be contributing to the speed oscillations whose amplitude given by $2/(a^2+ b^2)$ can be known by their residues. In this case as eigen values are complex conjugate their residues must be complex conjugate.

6.2 TRANSFER FUNCTION

Linearized model of induction machine can be represented by the following closed-loop diagram



Here P denotes derivative d/dt

$$G(p) = \Delta T_E/\Delta\theta \qquad (6.5)$$

Thus

$$\frac{\Delta\omega(p)}{\Delta T_L(p)} = \frac{\omega_e/2Hp}{1+\frac{\omega_e}{2Hp}\cdot\frac{G(p)}{p}} = \frac{p/\omega_e}{2Hp^2+\omega_e G(p)} \tag{6.6}$$

Lipo and Krause [8] give transfer function $\Delta T_e/\Delta\theta$ for synchronous machine. From this transfer function $\Delta T_e/\Delta\theta$ for induction machine is derived to be

$$G(p) = T_e/\Delta\theta = \frac{(V_1Z_4-V_2Z_2)V_3}{Z_1Z_4-Z_2Z_3} + \frac{(V_2Z_1 - V_1Z_3)\ V_4}{Z_1Z_4 - Z_2Z_3} \tag{6.7}$$

where $V_1 = -V\sin\theta_o + (p/\omega_e)(X_{ds}\ i_{dso})$

$$V_2 = V\cos\theta_o + (p/\omega_e)X_{qs}\ i_{qso}$$

$$V_3 = (X_{ds} - X_{qs})i_{dso} + \frac{(p/\omega_e)X_{aq}^2\ i_{dso}}{R_{KQ}+(p/\omega_e)\ X_{KQ}}$$

$$V_4 = (X_{ds} - X_{qs})\ i_{qso} - \frac{(p/\omega_e)X_{ad}^2\ i_{qso}}{R_{KD} + (p/\omega_e)X_{KD}}$$

$$Z_1 = R_s + (p/\omega_e)\ X_{Qs} - \frac{(p/\omega_e)^2\ X_{aq}{}^2}{R_{KQ}+(p/\omega_e)X_{KQ}}$$

$$Z_2 = f_R\ X_{ds} - (p/\omega_e) - \frac{(p/\omega_e)\ f_R\ X_{ad}{}^2}{R_{KD}+(p/\omega_e)X_{KD}}$$

$$Z_3 = -f_R\ Q_{qs} + \frac{(p/\omega_e)\ f_R\ X_{aq}{}^2}{R_{KQ}+(p/\omega_e)X_{KQ}}$$

$$Z_4 = R_s+(p/\omega_e)\ X_{ds} - \frac{(p/\omega_e)^2\ X_{ad}^{\ 2}}{R_{KD}+(p/\omega_e)X_{KD}}$$

$$r_r' \ = R_{KQ} = R_{KD}$$

$$X_s \ = X_{ds} = X_{QS}$$

$$X_r \ = X_{KD} = X_{KQ}$$

$$X_m \ = X_{ad} = X_{aq}$$

(r_r' , X_s , X_r and X_m are explained in Chapter 2).

6.3 Calculation of Residue

On substituting (6.7) in (6.6) we get

$$\Delta\omega(p)/\Delta T_L(p) = f_1(p)/f_2(p) \tag{6.8}$$

Poles of this transfer function are eigen values of the system matrix A. Thus $\Delta\omega(p)/\Delta T_L(p)$ can be written as

$$\Delta\omega(p)/\Delta T_L(p) = f_1(p)/[K\{(p+a)^2+ b^2\}(P+C)(p+j\omega_1)(p-j\omega_1)] \tag{6.9}$$

When the machine is running at the operating point which lies on the boundary of the region of instability, where $-a \pm jb$, $-c$ and $\pm j\omega_1$ are five eigen values of matrix $\omega_b A$ as given in Chapter 5.

$$f_1(p)/[K\{(p+a)^2+b^2\}(p+c)(p+j\omega_1)(p-j\omega_1)]$$

$$=[K_1/(p+a-jb)] + [K_2/(p+a+jb)] + [K_3/(p+c)]$$

$$+ [K_4/(p+j\omega_1)] + [K_5/(p-j\omega_1)] \tag{6.10}$$

Multiplying both sides by $(p - j\omega_1)$ and putting $p = j\omega_1$ we get

$$K_4 = f_1(j\omega_1)/[K\{(j\omega_1+a)^2+b^2\}(j\omega_1+c)(2j\omega_1)] \tag{6.11}$$

which can be solved to find out residue K_4.

From residue K_4 oscillation amplitude can be estimated.

steady state. These are nonlinear, algebraic equations which are solved by applying the ROSENBROCK Algorithm to obtain motor currents and speed in steady state. This analysis is carried out for an operating point where the machine is stable that is on a operating point which lies outside the region of instability. Using these steady state values as initial conditions, the non linear model of the machine is numerically integrated using Runge - Kutta fourth order method when the operating frequency is changed so that the new steady state operating point lies in the region of instability, to calculate motor currents and speed when it is unstable. Speed oscillations are observed here. To ensure that these speed oscillations are due to unstable mode of operation of the machine in this region the non linear model is again used to obtain mechanine performance when the frequency is again changed such that the new steady state point lies outside the region of instability. Initial condition for this part can be taken as currents and speed found at any time in the previous part. It was observed that machine settles to a new steady state point after the transients die out with no speed fluctuations.

The frequency of oscillation is also found from linearised model. For a critically stable system the frequency of oscillations is calculated from the imaginary part of the eigen value whose real part is equal to zero.

It is found by comparison that frequency computed by this method is almost equal to that found by simulating the non linear model for any operating point in the region of instability.

In the last chapter a short cut method is suggested to find out the amplitude of the oscillations from the linearised model. Here it is suggested that oscillation amplitude can be estimated within fair accuracy by studying the perturbation of speed around a steady state operating point.

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